The Economic Value of Volatility Timing Using a Range-based Volatility Model

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Abstract

There is growing interest in utilizing the range data of asset prices to study the role of volatility in financial markets. In this paper, we use a new range-based volatility model to examine the economic value of volatility timing in a mean-variance framework with three assets – stock, bond and cash. We compare its performance with a return-based dynamic volatility model in both in-sample and out-of-sample volatility timing strategies. For a risk-averse investor, it is shown that the predictable ability captured by the dynamic volatility models is economically significant, and that the range-based volatility model performs better than the return-based one. The results give robust inferences for supporting the range-based volatility model in forecasting volatility.

Keywords: Asset allocation, CARR, DCC, Economic value, Range, Volatility timing.

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I. Introduction

In recent years, there has been considerable interest in volatility. The extensive development of volatility modeling has been motivated by the related applications in risk management, portfolio allocation, assets pricing, and futures hedging. In discussions of econometric methodologies in estimating the volatility of individual assets, ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) have been emphasized most. Various applications in finance and economics are provided as a review in Bollerslev, Chou, and Kroner (1992), Bollerslev, Engle, and Nelson (1994), and Engle (2004).

Several studies, having noted that the range data based on the difference of high and low prices in a fixed interval, can offer a sharper estimate of volatility than the return data. A number of studies have investigated this issue, for example, Parkinson (1980), and more recently, Alizadeh, Brandt, and Diebold (2002), Brandt and Jones (2006) and Martens and Dijk (2007)[1]. Chou (2005) proposes a conditional autoregressive range (CARR) model which can easily capture the dynamic volatility structure and has obtained some satisfactory empirical results.

However, the literature above just focuses on volatility forecast of a univariate asset. It should be noted that there have been some attempts to establish a relationship between multiple assets, such as VECH (Bollerslev, Engle, and Wooldridge (1988)), BEKK (Engle and Kroner, 1995), and a constant conditional correlation model (CCC, proposed by Bollerslev, 1990), among others. VECH and BEKK allowing time-varying covariance process are too flexible to estimate, and CCC with a constant correlation is too restrictive to apply on general applications. Seminal work on solving

[1] See also Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1992), Yang and Zhang (2000), and Alizadeh, Brandt, and Diebold (2002).

the puzzle is carried out by Engle (2002a). A dynamic conditional correlation[2] (DCC) model proposed by Engle (2002a) provides another viewpoint to this problem. The estimation of DCC can be divided into two stages. The first step is to estimate univariate GARCH, and the second is to utilize the transformed standardized residuals to estimate time-varying correlations. See also Engle and Sheppard (2001) and Engle, Cappiello, and Sheppard (2006).

A new multivariate volatility, proposed by Chou, Liu, and Wu (2007), combines the range data of asset prices with the framework of DCC, namely range-based DCC[3]. They conclude that the range-based DCC model performs better than other return-based models (MA100, EWMA, CCC, return-based DCC, and diagonal BEKK) through the statistical measures, RMSE and MAE based on four benchmarks of implied and realized covariance[4].

Because the empirical results in many studies show that the forecast models only can explain little part of variations in time-varying volatilities, some studies are concentrated on whether volatility timing has economic value (Busse (1999), Fleming, Kirby, and Osdiek (2001,2003), Marquering and Verbeek (2004) and Thorp and Milunovich (2007)). The question we focus on is whether the economic value of volatility timing for range-based volatility model still exists and to test whether investors are willing to switch from a return-based DCC to a range-based DCC model.

For comparing the economic value, it is helpful to use a suitable measure to capture the trade-off between risk and return. Most literatures evaluate volatility models through error statistics and related applications, but neglect the influence of

[2] See Tsay (2002) and by Tse and Tsui (2002) for other related methods for estimating the time-varying correlations.
[3] Fernandes, Mota, and Rocha (2005) propose another kind of multivariate CARR model using the formula $Cov(X,Y)=[V(X+Y)-V(X)-V(Y)]/2$. However, this method can only apply to a bivariate case.
[4] Daily data are used to build four proxies for weekly covariances, i.e. implied return-based DCC, implied range-based DCC, implied DBEKK, and realized covariances.

asset expected returns. A more precise measurement should consider both of them, but only few studies have so far been made at this point. However, a utility function can easily connect them and build a comparable standard. Before entering into a detailed discussion for the economic value of volatility timing, it is necessary to clarify its definition in this paper. In short, the economic value of volatility timing is the gain compared with a static strategy. For an investor with a mean variance utility, our concern is to estimate his will to pay for a new volatility model rather than a static one.

In light of the success of the range-based volatility model, the purpose of this paper is to examine its economic value of volatility timing by using conditional mean-variance framework developed by Fleming, Kirby and Ostdiek (2001). We consider an investor with different risk-averse levels uses conditional volatility analysis to allocate three assets: stock, bond, and cash. Fleming, Kirby and Ostdiek (2001) extend West, Edison, and Cho's (1993) utility criterion to test the economic value of volatility timing for the short-horizon investors with different risk tolerance levels[5]. In addition, we also examine the economic value for longer horizon forecast of selected models in our empirical study. This study may lead to a better understanding of range volatility.

The remainder of this paper is laid out as follows. Section II introduces the asset allocation methodology, economic value measurement, and the return-based and the range-based DCC. Section III describes the properties of data used and evaluates the performance of the different strategies. Finally, the conclusion is showed in section IV.

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[5] They find that volatility-timing strategy based on one-step ahead estimates of the conditional covariance matrix (derived by Foster and Nelson (1996)) significantly outperformed the unconditional efficient static portfolios.

II. Methodology

The method to carry out this study is to use a framework of a minimum variance strategy, which is designed by a time-varying covariance[6]. For a risk-averse investor with one-day horizon based on this strategy, we want to find the optimal dynamic weights and the implied economic value. Before applying the volatility timing strategies, we need to build a time-varying covariance matrix. The Details of the methodology are as the following.

Optimal Portfolio Weights in a Minimum Variance Framework

Initially, we consider a minimization problem for the portfolio variance subjected to a target return constraint. To derive our strategy, we let the expected return of a $K \times 1$ vector of risky asset returns as $\boldsymbol{\mu} = E[\mathbf{R}_{t+1}]$[7] and its conditional covariance matrix as $\boldsymbol{\Sigma}_t = E_t[(\mathbf{R}_{t+1} - \boldsymbol{\mu})(\mathbf{R}_{t+1} - \boldsymbol{\mu})']$. That can be formulated as

$$\min_{w_t} \ \mathbf{w}_t' \boldsymbol{\Sigma}_t \mathbf{w}_t ,$$

$$s.t \ \ \mathbf{w}_t' \ \boldsymbol{\mu} + \left(1 - \mathbf{w}_t' \ \mathbf{1}\right) R_f = \mu_{t\arg et} , \qquad (1)$$

where $\mathbf{w}_t$ is a $K \times 1$ vector of portfolio weights for time *t*. R_f is the return for the risk-free asset. The optimal solution to the quadratic form (1) is:

$$\mathbf{w}_t = \frac{(\mu_{t\arg et} - R_f)\boldsymbol{\Sigma}_t^{-1}(\boldsymbol{\mu} - R_f \mathbf{1})}{(\boldsymbol{\mu} - R_f \mathbf{1})' \boldsymbol{\Sigma}_t^{-1}(\boldsymbol{\mu} - R_f \mathbf{1})} . \qquad (2)$$

It is clear to be expressed in a bivariate case ($K = 2$):

$$w_{1,t} = \frac{\dot{\mu}_{t\arg et}(\dot{\mu}_1 \sigma_{2,t}^2 - \dot{\mu}_2 \sigma_{12,t})}{\dot{\mu}_1^2 \sigma_{2,t}^2 + \dot{\mu}_2^2 \sigma_{1,t}^2 - 2\dot{\mu}_1 \dot{\mu}_2 \sigma_{12,t}} ,$$

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[6] Merton (1980) points out that the volatility process is more predictable than return series. In the in-sample comparison, we assume the expected returns are constant. However, in the out-of-sample comparison (one- to thirteen-steps ahead), the expected returns will vary with the sample rolling.

[7] Through out this paper, we use blackened letters to denote vectors or matrices.

$$w_{2,t} = \frac{\dot{\mu}_{t\arg et}(\dot{\mu}_2\sigma_{1,t}^2 - \dot{\mu}_1\sigma_{12,t})}{\dot{\mu}_1^2\sigma_{2,t}^2 + \dot{\mu}_2^2\sigma_{1,t}^2 - 2\dot{\mu}_1\dot{\mu}_2\sigma_{12,t}}, \quad (3)$$

where $\dot{\mu}_{t\arg et} = \mu_{t\arg et} - R_f$, $\dot{\mu}_1 = \mu_1 - R_f$, and $\dot{\mu}_2 = \mu_2 - R_f$ are the excess target returns and the excess spot returns of S&P 500 index (S&P 500) and 10-year Treasury bond (T-bond) in our empirical study. Under the cost of carry model, we can regard the excess returns as the futures returns by applying regular no-arbitrage arguments[8]. Furthermore, the covariance matrix $\mathbf{\Sigma}_t$ of the spot returns is the same as that of the excess (futures) returns.

Economic Value of Volatility Timing

Fleming, Kirby and Ostdiek (2001) use a generalization of West, Edison and Cho's (1993) criterion which builds the relation between a mean-variance framework and a quadratic utility to capture the trade-off between risk and return for ranking the performance of forecasting models. According to their work, the investor's utility can be defined as:

$$U(W_t) = W_t R_{p,t} - \frac{\alpha W_t^2}{2} R_{p,t}^2, \quad (4)$$

where W_t is the investor's wealth at time *t*, α is his absolute risk aversion, and the portfolio return at period *t* is $R_{p,t} = R_f + \mathbf{w}_t'\mathbf{R}_t$.

For comparisons across portfolios, we assume that the investor has a constant relative risk aversion (CRRA), $\gamma_t = \alpha W_t / (1 - \alpha W_t) = \gamma$. This implies αW_t is a constant. With this assumption, the average realized utility $\overline{U}(\cdot)$ can be used in estimating the expected utility with a given initial wealth W_0.

[8] There is no cost for futures investment. It means the futures return equals the spot return minus the risk-free rate.

$$\overline{U}(\cdot) = W_0 \sum_{t=1}^{T} \left[R_{p,t} - \frac{\gamma}{2(1+\gamma)} R_{p,t}^2 \right], \tag{5}$$

where W_0 is the initial wealth.

The value of volatility timing by equating the average utilities for two alternative portfolios is expressed as:

$$\sum_{t=1}^{T} \left[(R_{B,t} - \Delta) - \frac{\gamma}{2(1+\gamma)} (R_{B,t} - \Delta)^2 \right] = \sum_{t=1}^{T} \left[R_{A,t} - \frac{\gamma}{2(1+\gamma)} R_{A,t}^2 \right] ,$$

(6)

where Δ is the weekly maximum expenses that an investor would be willing to pay to switch from the strategy *A* to the strategy B. $R_{A,t}$ and $R_{B,t}$ here are the returns of the strategy *A* and *B*[9].

Return-based and Range-based DCC

We use the dynamic conditional correlation (DCC) model of Engle (2002a) to estimate the covariance matrix of multiple asset returns. It is a direct extension of the constant conditional correlation (CCC) model of Bollerslev (1990). The covariance matrix $\mathbf{H}_t$ for a vector of K asset returns in DCC can be written as:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{\Gamma}_t \mathbf{D}_t, \tag{7}$$

$$\mathbf{\Gamma}_t = diag\{\mathbf{Q}_t\}^{-1/2} \mathbf{Q}_t diag\{\mathbf{Q}_t\}^{-1/2}, \tag{8}$$

where, $\mathbf{D}_t$ is the $K \times K$ diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{k,t}}$ for the k^{th} return series on the k^{th} diagonal. $\mathbf{\Gamma}_t$ is a time-varying correlation matrix. Let $z_{k,t}$ be the standardized residual of the

9 In our setting, we let the strategy pair (*A*,*B*) be (OLS, return-based DCC), (OLS, range-based DCC), and (return-based DCC, range-based DCC), respectively. Because the rolling sample method is adopted in the out-of-sample comparison, we call this kind of OLS a new name, rollover OLS.

k^{th} asset, the covariance matrix $\mathbf{Q}_t = [q_{ij,t}]$ of standardized residual vector $\mathbf{Z}_t = (z_{1,t},....z_{K,t})'$ is governed by:

$$\mathbf{Q}_t = (1-a-b)\overline{\mathbf{Q}} + a\mathbf{Z}_{t-1}\mathbf{Z}'_{t-1} + b\mathbf{Q}_{t-1}, \tag{9}$$

where $\overline{\mathbf{Q}} = \{\overline{q}_{ij}\}$ denotes the unconditional covariance matrix of $\mathbf{Z}_t$. The coefficients, a and b, are parameters determining the evolution of the conditional correlations.

For a bivariate system, the dynamic correlation can be expressed as:

$$\rho_{12,t} = \frac{(1-a-b)\overline{q}_{12} + az_{1,t-1}z_{2,t-1} + bq_{12,t-1}}{\sqrt{[(1-a-b)\overline{q}_{11} + az^2_{1,t-1} + bq_{11,t-1}][(1-a-b)\overline{q}_{22} + az^2_{2,t-1} + bq_{22,t-1}]}}. \tag{10}$$

We can estimate the DCC model by the two-stage estimation through qusi-maximum likelihood estimation (QMLE) to get consistent parameter estimates. The log-likelihood function can expressed as $L = L_{Vol} + L_{Corr}$, where L_{Vol}, the volatility component, is $-\frac{1}{2}\sum_t \left(K\log(2\pi) + \log\left|\mathbf{D}_t\right|^2 + \mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{r}_t\right)$, and L_{Corr}, the correlation component, is $-\frac{1}{2}\sum_t \left(\log\left|\mathbf{R}_t\right| + \mathbf{Z}_t'\mathbf{R}_t^{-1}\mathbf{Z}_t - \mathbf{Z}_t'\mathbf{Z}_t\right)$. The explanation is more fully developed in Engle (2002), and Chou, Liu, and Wu (2007).

In addition to using GARCH to construct standardized residuals, we can also build them by other univariate volatility models. In this paper, we use the CARR model of Chou (2005) as an alternative for the specification for volatilities..

The CARR model is a special case of the multiplicative error model (MEM) of Engle (2002b). It can be expressed as:

$$\Re_{k,t} = \lambda_{k,t}u_{k,t}, \quad u_{k,t} \mid I_{t-1} \sim \exp(1,\cdot), \quad k = 1, 2,$$

$$\lambda_{k,t} = \omega_k + \alpha_k \Re_{k,t-1} + \beta_k \lambda_{k,t-1}, \tag{11}$$

$$z^c_{k,t} = r_{k,t} / \lambda^*_{k,t} \text{, where } \lambda^*_{k,t} = adj_k \times \lambda_{k,t} \text{, } adj_k = \frac{\overline{\sigma}_k}{\hat{\overline{\lambda}}_k},$$

where the range $\Re_{k,t}$ is calculated by the difference between logarithm high and low prices of the k^{th} asset during a fixed time interval t and it is also a proxy of standard deviation. $\lambda_{k,t}$ and $\overline{\hat{\lambda}_k}$ are the conditional and unconditional mean of the range, respectively. $u_{k,t}$ is the residual which is assumed to follow the exponential distribution. $\overline{\sigma}_k$ is the unconditional standard deviation for the return series. Considering different scales in quantity, the ratio adj_k is used to adjust the range to produce the standardized residuals.

III. Empirical Results

The empirical data employed in this paper consist of the stock index futures, bond futures and the risk-free rate. As to the above-mentioned method, we apply the futures data to examine the economic value of volatility timing for return-based and range-based DCC. Under the cost of carry model, the result in this case can be extended to underlying spot assets (Fleming, Kirby, and Osdiek (2001)). In addition to avoiding the short sale constraints, this procedure will reduce the complexity of model setting. To address this issue, we use the S&P 500 futures (traded at CME), and the T-bond futures (traded at CBOT) as the empirical samples. The futures data is taken from Datastream, sampling from January 6, 1992 to December 29, 2006 (782 weekly observations). Datastream provides the nearest contract and rolls over to the second nearby contract when the nearby contract approaches maturity. We also use the 3-month Treasury bill rate to substitute for the risk-free rate. The Treasury bill rate is available in the Federal Reserve Board.

< Figure 1 is inserted about here >

Figure 1 shows the graphs for close prices (Panel A), returns (Panel B) and ranges (Panel C) of the S&P 500 and T-bond futures over the sample period. Table 1 shows summary statistics for the return and range data on the S&P 500 and T-bond futures. The return is computed as the difference of logarithm close prices on two continuous weeks. The range is defined by the difference of the high and low prices in a logarithm type. The annualized mean and standard deviation in percentage, (8.210, 15.232) of the stock futures returns are both larger than those (0.583, 6.168) of the bond futures returns. The fact indicates that the more volatile market may have higher risk premium. Both futures returns have negative skewness and excess kurtosis, indicating violation of the normal distribution. The range mean (3.134) of the stock futures prices is larger than that (1.306) of the bond futures prices. It is reasonable because the range is a proxy of volatility. The Jarque-Bera statistic is used to test the null of whether the return and range data are normally distributed. Undoubtedly, both of return and range data reject the null hypothesis. The simple correlation between stock and bond returns is small[10] (-0.023), but it does not imply that their relation is very weak. In our latter analysis, we show that the dynamic relationship of stock and bond will be more realistically revealed by the conditional correlations analysis.

< Table 1 is inserted about here >

The In-sample Comparison

For obtaining an optimal portfolio, we use the dynamic volatility models to estimate the covariance matrices. As for the parameters fitted for return-based and range-based DCC, they are both estimated and arranged in Table 2. We divide the table into two parts corresponding to the two steps in the DCC estimation. In Panel A, one can use GARCH (fitted by return) or CARR (fitted by range) with individual assets to obtain

[10] The result is different from the positive correlation value (sample period 1983-1997) in Fleming, Kirby, and Osdiek (2001). About after 1997, the relationship between S&P 500 and T-bond presents a reverse condition.

the standardized residuals. Figure 2 provides the volatility estimated of the S&P 500 futures and the T-bond futures based on GARCH and CARR. Then, these standardized residuals series can be brought into the second stage for dynamic conditional correlation estimating. Panel B shows the estimated parameters of DCC under the quasi-maximum likelihood estimation (QMLE).

< Table 2 is inserted about here >

< Figure 2 is inserted about here >

The correlation and covariance estimates for return-based and range-based DCC are shown in Figure 3. It seems that the correlation becomes more negative at the end of 1997. A deeper investigation is given in Connolly, Stivers, and Sun (2005).

< Figure 3 is inserted about here >

Following the model estimation, we construct the static portfolio (built by OLS) using the unconditional mean and covariance matrices for getting the economic value of dynamic models. Under the minimum variance framework, the weights of the portfolio are computed by the given expected return[11] and the conditional covariance matrices estimated by return-based and range-based DCC. Then, we want to compare the performance of the volatility models on 11 different target annualized returns (5% - 15%, 1% in an interval) .

< Table 3 is inserted about here >

Table 3 shows how the performance comparisons vary with the target returns and the risk aversions. Panel A shows the annualized means (μ) and volatilities (σ) of the portfolios estimated from three methods, return-based DCC, range-based DCC, and OLS. For a quick look, the annualized Sharpe ratios[12] calculated from return-based DCC (0.680) and range-based DCC (0.699) are higher than the static

[11] Also see footnote 5 for a reference.
[12] The Sharpe ratio is constant with different target multipliers.

model (0.560). Panel B shows the average switching fees (Δ_r) from one strategy to another. The value setting of CRRA γ are 1, 5, and 10. As for the performance fees with different relative risk aversions, in general, an investor with a higher risk aversion would be willing to pay more to switch from the static portfolio to the dynamic ones. With higher target returns, the performance fees are increasing steadily. In addition, Panel B also reports the performance fees switching from return-based DCC to range-based DCC. Positive values for all cases show that the range-based volatility model can give significant economic value in forecasting covariance matrices than the return-based ones. Figure 4 plots the weights of in-sample minimum volatility portfolio derived from two dynamic models. In the meanwhile, OLS has constant weights for cash, stock, and bond, i.e. -0.1934, 0.7079, and 0.4855.

< Figure 4 is inserted about here >

The Out-of-sample Comparisons

For robust inference, a similar approach is utilized to estimate the value of volatility timing in the out-of-sample analysis. Here the rolling sample approach is adopted for all out-of-sample estimations. It means that the rollover OLS method replaces the conventional OLS method which is used in the in-sample analysis. Each forecasting value is estimated by 521 observations, about 10 years. Then, the rolling sample method provides 249 forecasting values for one to thirteen-period ahead comparisons. The first forecasted value occurs the week of January 4, 2002.

< Table 4 is inserted about here >

Table 4 reports how the performance comparisons vary with the target returns and the risk aversions for one period ahead out-of-sample forecast. We obtain a consistent conclusion with Table 3. The estimated Sharpe ratios calculated from return-based DCC, range-based DCC, and rollover OLS are 0.504, 0.452, and 0.251, respectively. The performance fees switching from rollover OLS to DCC are all positive. In total, the out-of-sample comparison supports the former inference. Figure 5 plots the weights that minimize conditional volatility while setting the expected annualized return equal to 10%.

< Figure 5 is inserted about here >

Table 5 reports one to thirteen periods ahead out-of-sample performance for three methods. The portfolio weights for all strategies are obtained from the weekly estimates of the one to thirteen periods ahead conditional covariance matrices with a fixed target return (10%). Roughly speaking, the Sharpe ratios got from range-based DCC is the largest, and return-based DCC is the next. For each strategy, however, we can not find an obvious trend in the Sharpe ratios with forecasting periods ahead. As for the result of the performance fees, it seems reasonable to conclude that an investor is still willing to pay to switch from rollover OLS to DCC. Moreover, the economic value seems to appear a decreasing trend with forecasting periods ahead. For longer forecasting horizon (11-13 weeks), however, the result of estimated economic value for the two dynamic models is not clear. As to the will switching from return-based DCC to range-based DCC, it always keeps positive.

< Table 5 is inserted about here >

IV. Conclusion

In this paper, we examine the economic value of volatility timing for the range-based volatility model in utilizing the range data which combines CARR with a DCC structure. Applying S&P 500 and T-bond futures to a mean-variance framework with a no-arbitrage setting, the result can be extended to spot asset analysis. By means of the utility of portfolio, the economic value of dynamic models can be obtained from comparing with OLS. Both of in-sample and out-of-sample results show that a risk-averse investor is willing to switch from OLS to DCC. Moreover, the switching fees from return-based DCC to range-based DCC are always positive. We can conclude that the range-based volatility model has more significant economic value compared to the return-based one. The results are entirely consistent with those reported in Chou, Liu, and Wu (2007).

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Table 1: Summary Statistics for Weekly S&P 500 and T-bond Futures Return and Range Data, 1992-2006

The table provides summary statistics for the weekly return and range data on S&P 500 stock index futures and T-bond Futures. The returns and ranges are computed by $100\times\log(p_t^{close}/p_{t-1}^{close})$ and $100\times\log(p^{high}/p^{low})$, respectively. The *Jarque-Bera* statistic is used to test the null of whether the return and range data are normally distributed. The values presented in parentheses are p-values. The annualized values of means (standard deviation) for S&P 500 and T-bond futures are 8.210 (15.232) and 0.583 (6.168), respectively. The simple correlation between stock and bond returns is -0.023. The sample period ranges form January 6, 1992 to December 29, 2006 (15 years, 782 observations) and all futures data are collected from Datastream.

	S&P 500 Futures		T-Bond Futures	
	Return	Range	Return	Range
Mean	0.158	3.134	0.016	1.306
Median	0.224	2.607	0.033	1.194
Maximum	8.124	13.556	2.462	4.552
Minimum	-12.395	0.690	-4.050	0.301
Std. Dev.	2.112	1.809	0.855	0.560
Skewness	-0.503	1.756	-0.498	1.390
Kurtosis	6.455	7.232	4.217	6.462
Jarque-Bera	421.317	985.454	80.441	642.367
	(0.000)	(0.000)	(0.000)	(0.000)

Table 2: Estimation Results of Return-based and Range-based DCC Model Using Weekly S&P500 and T-bond Futures, 1992-2006

$$r_{k,t} = c + \varepsilon_{k,t}\,,\ \ h_{k,t} = \omega_k + \alpha_k \varepsilon^2_{k,t-i} + \beta_k h_{k,t-1}\,,\ \ \varepsilon_{k,t} \mid I_{t-1} \sim N(0, h_{k,t})\,,$$

$$\Re_{k,t} = \lambda_{k,t} u_{i,t}\,,\ \ \lambda_{k,t} = \omega_k + \alpha_k \Re_{k,t-1} + \beta_k \lambda_{k,t-1}\,,\ \ \Re_{k,t} \mid I_{t-1} \sim exp(1,\cdot)\,,\ \ k = 1,2\,.$$

$$\mathbf{Q}_t = (1-a-b)\overline{\mathbf{Q}} + a\mathbf{Z}_{t-1}\mathbf{Z}'_{t-1} + b\mathbf{Q}_{t-1}, \text{ and then}$$

$$\rho_{12,t} = \frac{(1-a-b)\bar{q}_{12} + az_{1,t-1}z_{2,t-1} + bq_{12,t-1}}{\sqrt{[(1-a-b)\bar{q}_{11} + az^2_{1,t-1} + bq_{11,t-1}][(1-a-b)\bar{q}_{22} + az^2_{2,t-1} + bq_{22,t-1}]}}\,,$$

where $\Re_t$ is the range variable, $\mathbf{Z}_t$ is the standard residual vector which is standardized by GARCH or CARR volatilities. $\mathbf{Q}_t = \{q_{ij,t}\}$ and $\overline{\mathbf{Q}} = \{\bar{q}_{ij}\}$ are the conditional and unconditional covariance matrix of $\mathbf{Z}_t$. The three formulas above are GARCH, CARR and the conditional correlation equations respectively of the standard DCC model with mean reversion. The table shows estimations of the three models using the MLE method. Panel A is the first step of the DCC model estimation. The estimation results of GARCH and CARR models for two futures are presented here. $Q(12)$ is the *Ljung–Box* statistic for the autocorrelation test with 12 lags. Panel B is the second step of the DCC model estimation. The values presented in parentheses are t-ratios for the model coefficients and p-values for Q(12).

Panel A: Volatilities Estimation of GARCH and CARR models				
	S&P500 Futures		T-bond Futures	
	GARCH	CARR	GARCH	CARR
c	0.188		0.008	
	(3.256)		(0.242)	
$\hat{\omega}$	0.019	0.103	0.028	0.075
	(1.149)	(2.923)	(1.533)	(2.810)
$\hat{\alpha}$	0.051	0.248	0.060	0.157
	(3.698)	(9.090)	(2.031)	(5.208)
$\hat{\beta}$	0.946	0.719	0.902	0.785
	(71.236)	(23.167)	(18.645)	(18.041)
Q(12)	26.322	5.647	15.872	23.121
	(0.010)	(0.933)	(0.197)	(0.027)

Panel B: Correlation Estimation of Return- and Range-based DCC Models		
	S&P500 and T-bond	
	Return-based DCC	Range-based DCC
$\hat{a}$	0.037	0.043
	(4.444)	(4.679)
$\hat{b}$	0.955	0.951
	(85.621)	(80.411)

Table 3: In-sample Comparison of the Volatility Timing Values in the Minimum Volatility Strategy Using Different Target Returns, 1992-2006

The table reports the in-sample performance of the volatility timing strategies with different target returns. The target returns are from 5% to 15% (annualized). The weights for the volatility timing strategies are obtained from the weekly estimates of the conditional covariance matrix and the different target return setting. Panel A shows the annualized means (μ) and volatilities (σ) for each strategy. The estimated Sharpe ratios for the return-based DCC model, the range-based DCC model, and the OLS strategy are 0.680, 0.699, and 0.560, respectively. Panel B shows the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion γ are 1, 5, and 10.

Panel A: Means and Volatilities of Optimal Portfolios

Target	Return-based DCC		Range-based DCC		OLS	
return(%)	μ	σ	μ	σ	μ	σ
5	5.201	2.100	5.241	2.100	5.000	2.190
6	6.366	3.814	6.438	3.813	6.000	3.977
7	7.530	5.527	7.635	5.526	7.000	5.764
8	8.694	7.241	8.832	7.239	8.000	7.551
9	9.859	8.954	10.028	8.952	9.000	9.338
10	11.023	10.668	11.225	10.665	10.000	11.125
11	12.187	12.381	12.422	12.378	11.000	12.912
12	13.352	14.095	13.619	14.091	12.000	14.699
13	14.516	15.808	14.815	15.804	13.000	16.486
14	15.680	17.521	16.012	17.517	14.000	18.273
15	16.845	19.235	17.209	19.230	15.000	20.060

Panel B: Switching Fees with Different Relative Risk Aversions

Target	OLS to Return DCC			OLS to Range DCC			Return to Range DCC		
return(%)	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}
5	0.303	0.376	0.393	0.343	0.417	0.434	0.040	0.041	0.041
6	0.703	0.950	1.008	0.777	1.025	1.084	0.074	0.076	0.076
7	1.244	1.771	1.897	1.353	1.883	2.009	0.109	0.112	0.112
8	1.929	2.845	3.063	2.073	2.994	3.213	0.144	0.149	0.151
9	2.761	4.173	4.507	2.940	4.360	4.696	0.180	0.189	0.191
10	3.739	5.753	6.224	3.956	5.979	6.453	0.217	0.230	0.233
11	4.866	7.578	8.206	5.121	7.846	8.477	0.255	0.273	0.277
12	6.142	9.641	10.441	6.434	9.951	10.754	0.294	0.318	0.324
13	7.565	11.932	12.914	7.897	12.283	13.270	0.334	0.365	0.373
14	9.135	14.436	15.609	9.507	14.831	16.009	0.375	0.414	0.424
15	10.851	17.142	18.509	11.262	17.580	18.952	0.418	0.466	0.479

Table 4: Out-of-sample Comparison for the One Period Ahead Volatility Timing Values in the Minimum Volatility Strategy with Different Target Returns, 1992-2006

The table reports the one period ahead out-of-sample performance of the volatility timing strategies with different target returns. There are 521 observations in each of the estimated models and the rolling sample approach provides 249 forecasting values for each out-of-sample comparison. The first forecasted value occurs the week of January 4, 2002. The target returns are from 5% to 15% (annualized). The weights for the volatility timing strategies are obtained from the weekly estimates of the one period ahead conditional covariance matrix and the different target return setting. Panel A shows the annualized means (μ) and volatilities (σ) for each strategy. The estimated Sharpe ratios for the return-based DCC model, the range-based DCC model, and the rollover OLS strategy are 0.504, 0.452, and 0.251, respectively. Panel B shows the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion are 1, 5, and 10.

Panel A: Means and Volatilities of Optimal Portfolios

Target	Return-based DCC		Range-based DCC		Rollover OLS	
return(%)	μ	σ	μ	σ	μ	σ
5	4.551	1.718	4.622	1.682	4.217	1.763
6	5.184	3.119	5.315	3.054	4.578	3.202
7	5.817	4.520	6.007	4.426	4.939	4.640
8	6.450	5.921	6.699	5.798	5.300	6.079
9	7.084	7.323	7.391	7.170	5.661	7.517
10	7.717	8.724	8.083	8.542	6.022	8.956
11	8.350	10.125	8.775	9.914	6.383	10.394
12	8.984	11.527	9.467	11.286	6.744	11.833
13	9.617	12.928	10.159	12.658	7.105	13.271
14	10.250	14.329	10.851	14.030	7.466	14.710
15	10.884	15.730	11.543	15.402	7.827	16.148

Panel B: Switching Fees with Different Relative Risk Aversions

Target	OLS to Return DCC			OLS to Range DCC			Return to Range DCC		
return(%)	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}
5	0.375	0.404	0.411	0.478	0.530	0.543	0.104	0.126	0.132
6	0.742	0.840	0.863	0.977	1.150	1.190	0.236	0.312	0.330
7	1.165	1.372	1.420	1.574	1.937	2.022	0.412	0.574	0.613
8	1.644	2.000	2.084	2.270	2.892	3.037	0.632	0.914	0.982
9	2.180	2.725	2.853	3.064	4.011	4.232	0.897	1.334	1.438
10	2.772	3.546	3.727	3.955	5.293	5.601	1.207	1.834	1.984
11	3.420	4.463	4.705	4.945	6.731	7.139	1.564	2.416	2.619
12	4.125	5.473	5.785	6.031	8.321	8.838	1.967	3.079	3.344
13	4.887	6.575	6.964	7.213	10.055	10.689	2.416	3.825	4.160
14	5.704	7.768	8.239	8.488	11.926	12.684	2.914	4.653	5.064
15	6.578	9.047	9.607	9.857	13.928	14.813	3.458	5.562	6.057

Table 5: Out-of-sample Comparison for One to Thirteen Periods Ahead Volatility Timing Values in the Minimum Volatility Strategy, 1992-2006

The table reports the one to thirteen periods ahead out-of-sample performance of the volatility timing strategies with the fixed 10% (annualized) target return. The weights for the volatility timing strategies are obtained from the weekly estimates of the one to thirteen periods ahead conditional covariance matrix. There are 521 observations in each of the estimated models and the rolling sample approach provides 249 forecasting values for each out-of-sample comparison. The first forecasted mean value occurs the week of January 4, 2002. Panel A shows the annualized means (μ), volatilities (σ), and Sharpe ratios (SR) for each strategy. Panel B shows the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion are 1, 5, and 10.

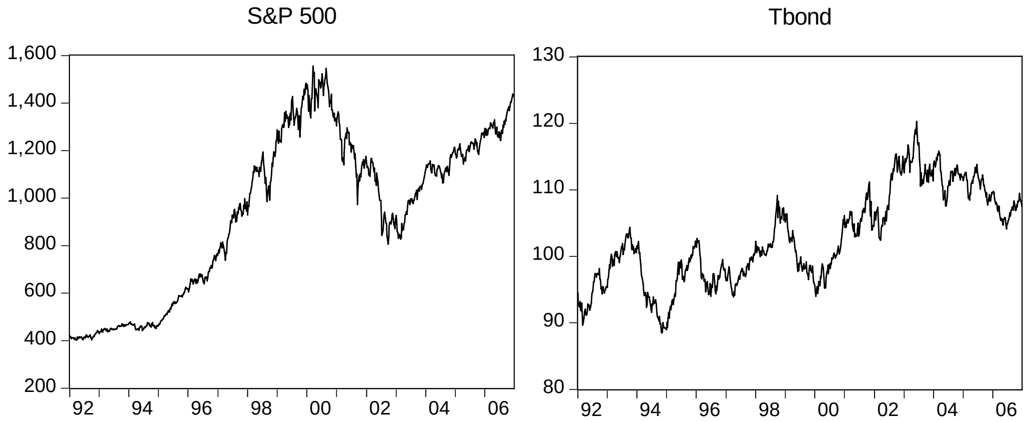
Panel A: Means and Volatilities of Optimal Portfolios

Periods Ahead	Return-based DCC			Range-based DCC			Rollover OLS		
	μ	σ	SR	μ	σ	SR	μ	σ	SR
1	7.717	8.724	0.452	8.083	8.542	0.504	6.022	8.956	0.251
2	7.868	8.830	0.464	8.477	8.559	0.549	6.068	8.933	0.257
3	7.371	8.807	0.408	8.318	8.584	0.529	6.660	8.931	0.323
4	8.117	8.838	0.491	8.736	8.605	0.577	7.103	8.928	0.373
5	8.464	8.860	0.529	9.071	8.664	0.611	6.869	8.989	0.344
6	9.088	8.903	0.597	9.565	8.664	0.668	7.232	8.973	0.385
7	9.361	8.840	0.632	9.967	8.634	0.717	7.872	8.945	0.458
8	8.853	8.897	0.571	9.388	8.704	0.645	7.644	8.975	0.431
9	9.806	8.878	0.679	10.022	8.714	0.717	8.476	9.023	0.521
10	9.746	8.887	0.672	9.517	8.716	0.659	8.189	8.983	0.491
11	9.436	8.908	0.636	8.899	8.700	0.589	8.031	8.910	0.478
12	8.737	9.003	0.551	7.962	8.816	0.475	7.424	8.853	0.412
13	8.713	9.111	0.542	8.277	8.939	0.504	7.794	8.867	0.453

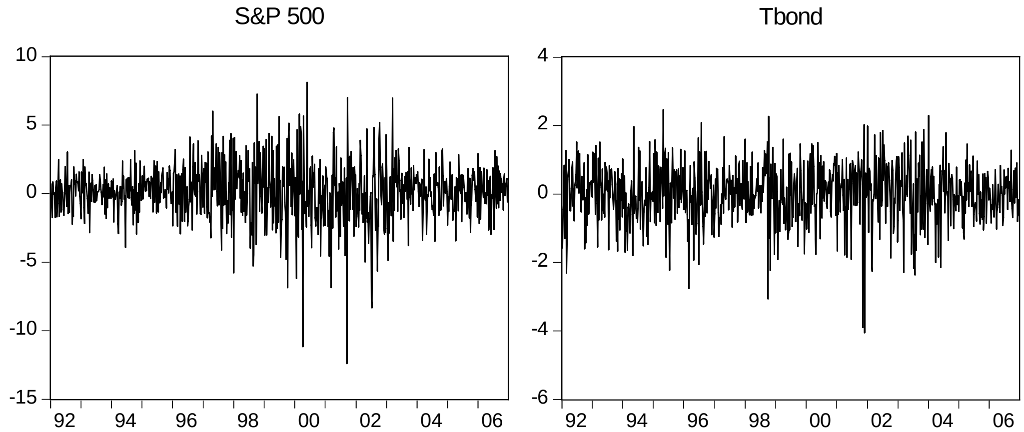
Panel B: Switching Fees with Different Relative Risk Aversions

Periods Ahead	OLS to Return DCC			OLS to Range DCC			Return to Range DCC		
	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}
1	2.772	3.546	3.727	3.955	5.293	5.601	1.207	1.834	1.984
2	2.282	2.633	2.716	4.126	5.344	5.625	1.874	2.809	3.030
3	1.293	1.721	1.823	3.263	4.416	4.684	1.984	2.748	2.929
4	1.440	1.758	1.834	3.136	4.226	4.482	1.712	2.526	2.720
5	2.210	2.665	2.773	3.720	4.817	5.073	1.531	2.226	2.393
6	2.191	2.442	2.503	3.783	4.839	5.087	1.617	2.481	2.689
7	1.993	2.373	2.464	3.558	4.635	4.889	1.585	2.334	2.515
8	1.581	1.861	1.928	3.022	3.962	4.185	1.456	2.154	2.322
9	2.028	2.556	2.683	3.018	4.113	4.372	1.007	1.620	1.770
10	2.019	2.370	2.455	2.596	3.539	3.763	0.593	1.230	1.385
11	1.416	1.424	1.426	1.858	2.597	2.773	0.457	1.217	1.401
12	0.593	0.037	-0.100	0.713	0.845	0.877	0.126	0.809	0.973
13	-0.269	-1.202	-1.436	0.139	-0.125	-0.189	0.406	1.045	1.199

Panel A: Close Prices



Panel B: Returns



Panel C: Ranges

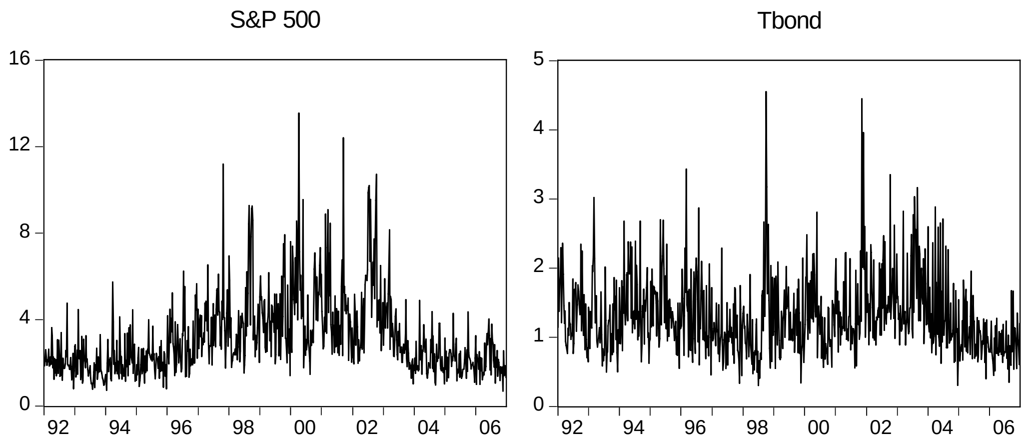
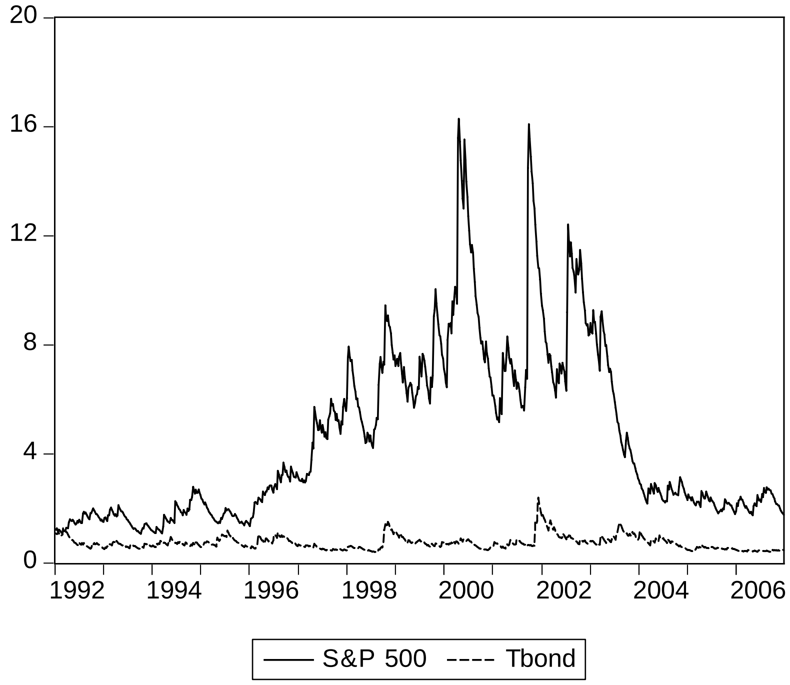


Figure 1: S&P 500 Index Futures and T-bond Futures Weekly Closing Prices, Returns and Ranges, 1992-2006. This figure shows the weekly close prices, returns, and ranges of S&P 500 index futures and 10-year Treasury bond (T-bond) futures over the sample period.

Panel A: Volatility Estimates for the GARCH Model



Panel B: Volatility Estimates for the CARR Model

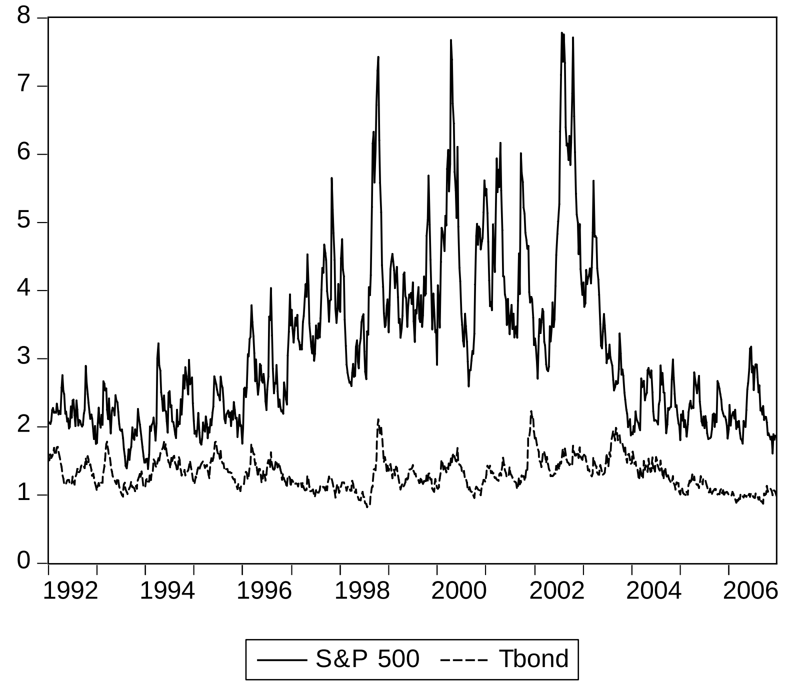
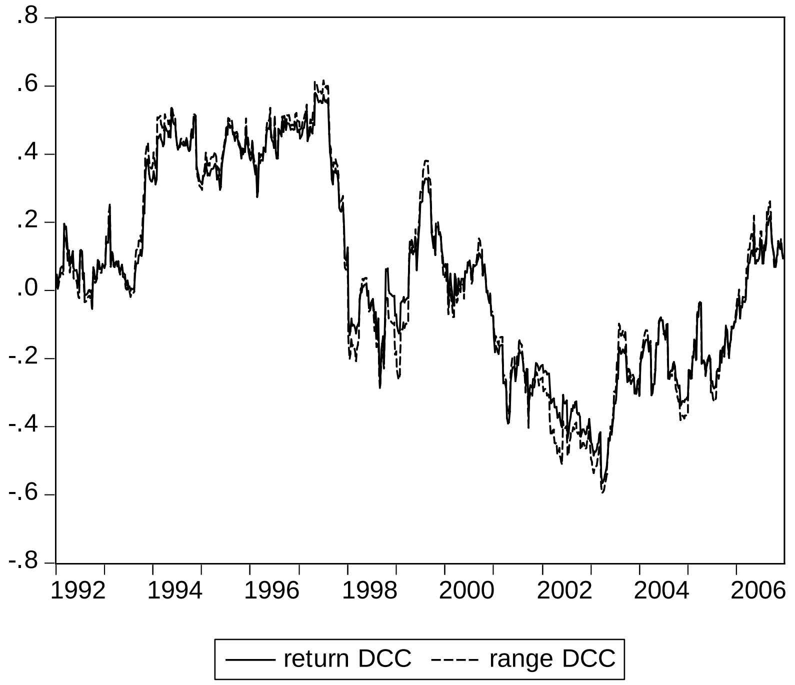


Figure 2: In-sample Volatility Estimates for the GARCH and CARR Model

Panel A: Correlation Estimates



Panel B: Covariance Estimates

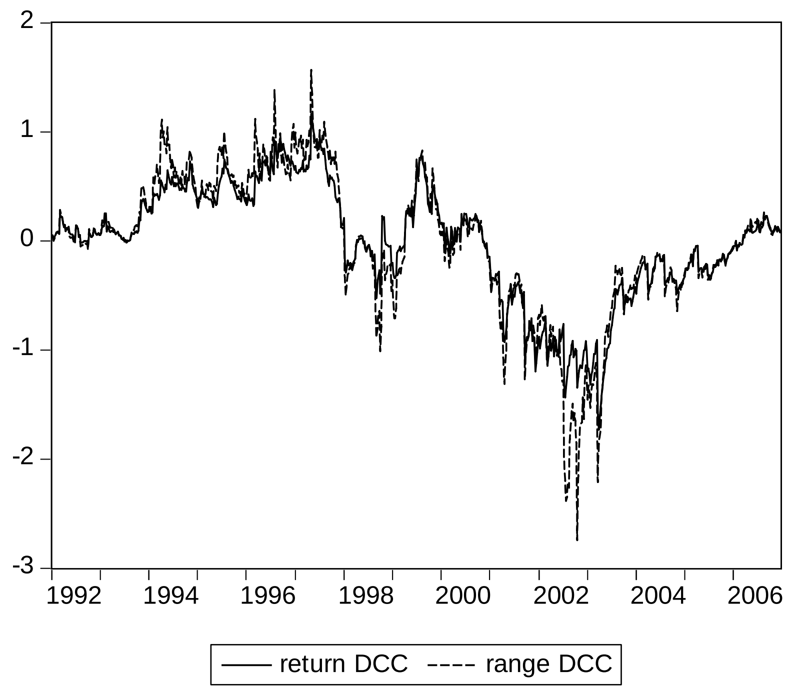
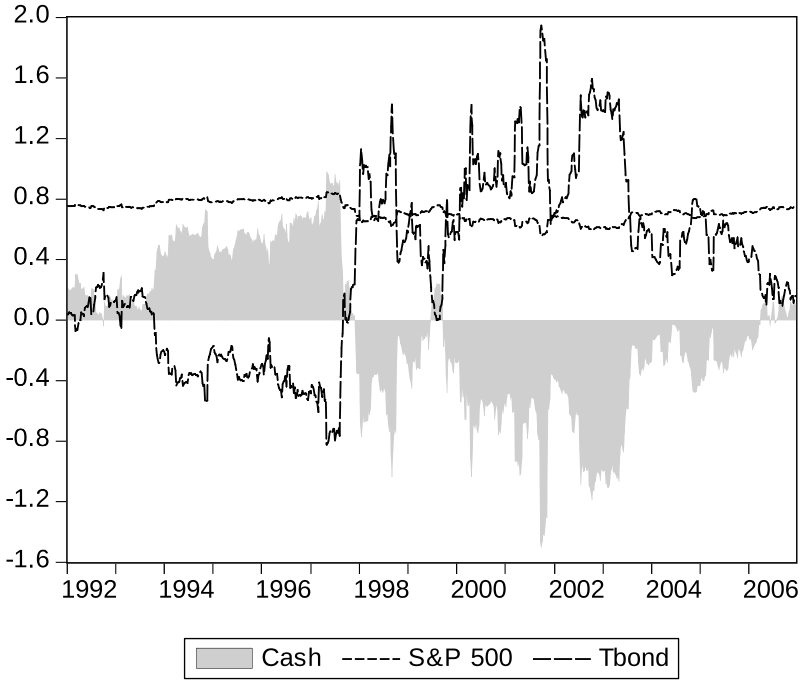


Figure 3: In-sample Correlation and Covariance Estimates for the Return-based and Range-based DCC Model

Panel A: In-sample Portfolio Weights Derived by the Return-based DCC Model



Panel B: In-sample Portfolio Weights Derived by the Range-based DCC Model

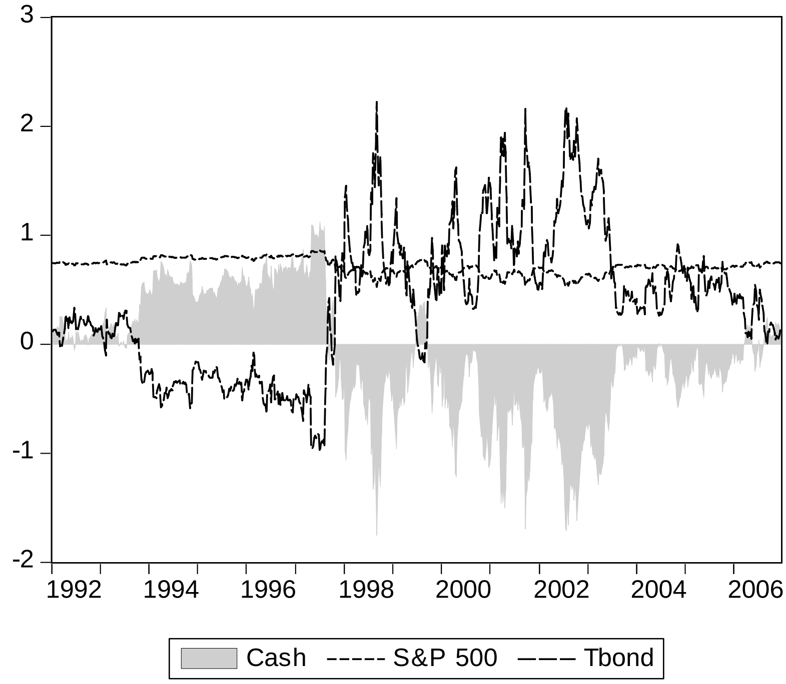
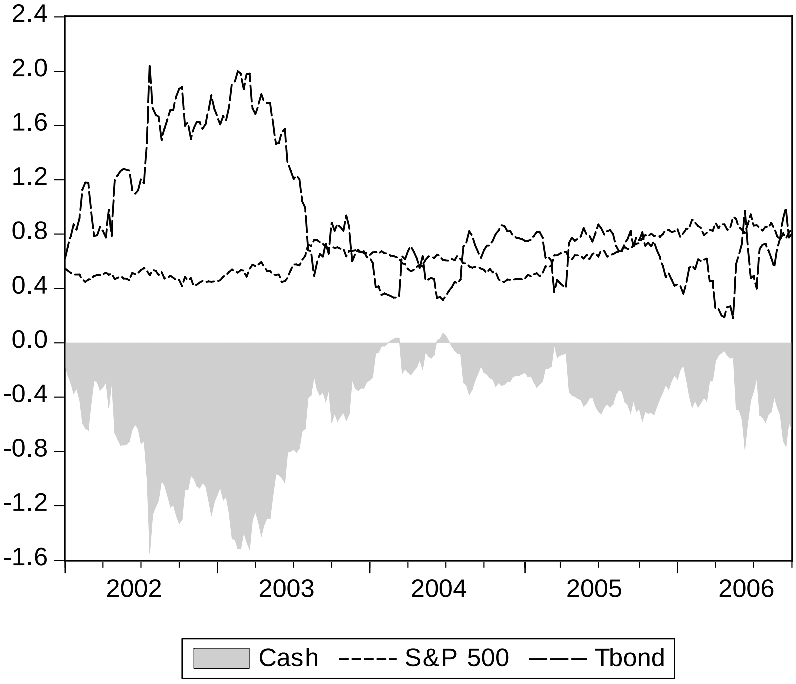
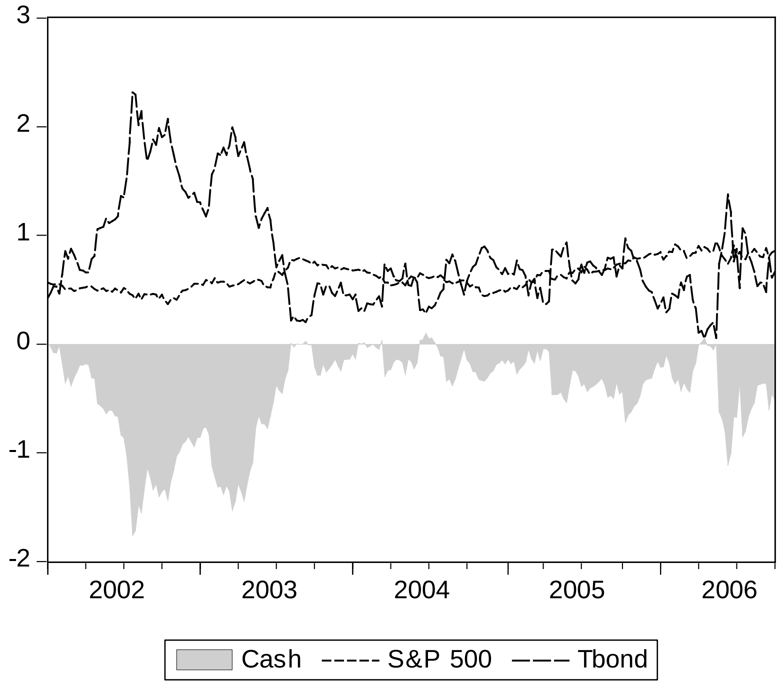


Figure 4: In-sample Minimum Volatility Portfolio Weight Derived by the Dynamic Volatility Model. Panels A and B show the weights that minimize conditional volatility while setting the expected annualized return equal to 10%. The OLS model has constant weights for cash, stock, and bond, i.e. -0.1934, 0.7079, and 0.4855.

Panel A: Out-of-sample Portfolio Weight Derived by the Return-based DCC Model



Panel B: Out-of-sample Portfolio Weight Derived by the Range-based DCC Model



Panel C: Out-of-sample Portfolio Weight Derived by the Rollover OLS Model

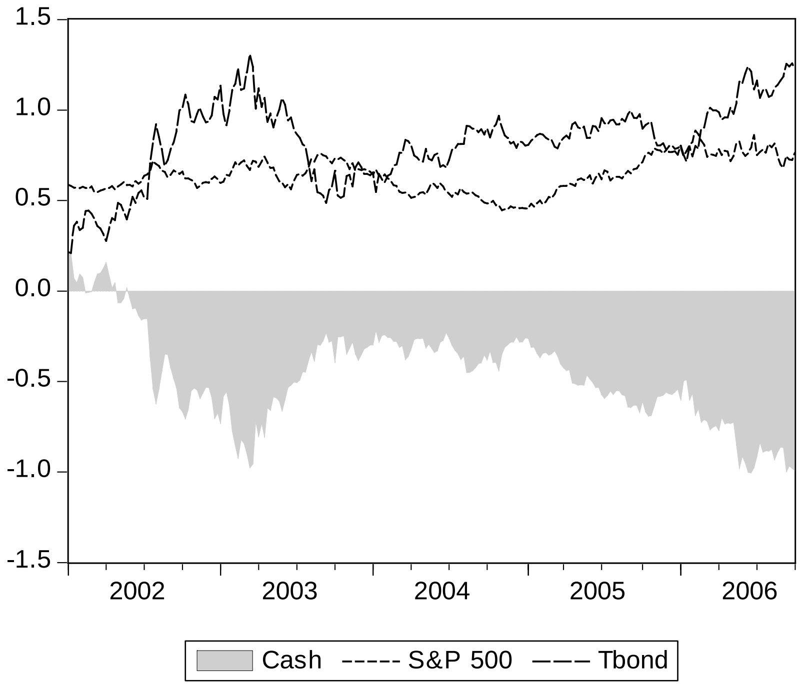


Figure 5: Out-of-sample Minimum Volatility Portfolio Weight Derived by the Dynamic Volatility Model for One Period Ahead Estimates. Panels A, B, and C show the one period ahead weights that minimize conditional volatility while the expected annualized return equal is set to 10%. Different from the in-sample case, the rolling sample method is used in the portfolio weights estimation. The portfolio weights in the rollover OLS model (Panel C) also vary with time. The first forecasted weights occur the week of January 4, 2002.