Inflation's Role in Optimal Monetary-Fiscal Policy*

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Abstract

We study how the maturity structure of nominal government debt affects optimal monetary and fiscal policy decisions and equilibrium outcomes in the presence of distortionary taxes and sticky prices. Key findings include: (1) there is always a role for current and future inflation innovations to revalue government debt, reducing reliance on distorting taxes; (2) the role of inflation in optimal fiscal financing increases with the average maturity of government debt; (3) as average maturity rises, it is optimal to tradeoff inflation for output stabilization; (4) inflation is relatively more important as a fiscal shock absorber in high-debt than in low-debt economies; (5) in some calibrations that are relevant to U.S. data, welfare under the fully optimal monetary and fiscal policies can be made equivalent to the welfare under the conventional optimal monetary policy with passively adjusting lump-sum taxes.

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1 Introduction

Many countries have adopted monetary and fiscal policy arrangements that erect firm walls between the two policy authorities. There are good practical reasons for this separation: historically, high- or hyperinflation episodes have sprung from governments pressuring central banks to finance spending by printing high-powered money. Economic theory does not uniformly support the complete separation. If inflation is costless, as in neoclassical models with flexible wages and prices, then Chari and Kehoe (1999) show that an optimal policy generates jumps in inflation that revalue nominal government debt without requiring changes in distorting tax rates, much as inflation behaves under the fiscal theory of the price level [Leeper (1991), Sims (1994), Woodford (1995)].

Schmitt-Grohé and Uribe (2004) and Siu (2004) overturn this role for inflation with the striking result that even a modicum of price stickiness makes the optimal volatility of inflation close to zero, an outcome later confirmed by Kirsanova and Wren-Lewis (2012), among others. Out of this optimal policy literature has emerged the "current consensus assignment" for monetary and fiscal policy, which Kirsanova et al. (2009) articulate: give monetary policy the task of controlling demand and inflation and fiscal policy the job of stabilizing debt. Actual policy arrangements in most countries are consistent with this literature's conclusions.

Sims (2001, 2013) questions whether the consensus assignment is robust when governments issue long-term nominal bonds. He lays out a theoretical argument for using nominal debt—and surprise revaluations of that debt—as a cushion against fiscal shocks to substitute for large movements in distorting taxes. Turning to data, Sims calculates that in the United States surprise gains and losses on debt due to inflation are on the order of 6 percent of the value of outstanding debt, roughly the magnitude of fluctuations in primary surpluses. Sims (2013) stops short of claiming that the significant responses of inflation to fiscal disturbances, which long debt permits, is optimal policy.

This paper explores that claim. In particular, we extend Sims's reasoning to the canonical new Keynesian model that Benigno and Woodford (2004, 2007) examine. The steady state is distorted by monopolistically competitive firms and nominal rigidities prevent firms from choosing new prices each period. A distorting tax is levied against firms' sales. Total factor productivity, wage markups, government purchases of goods, and lump-sum government transfers fluctuate exogenously. Government issues nominal debt whose average duration is indexed by a single parameter. Monetary and tax policies are chosen optimally to maximize welfare of the representative household. Optimal policies and the nature of resulting equilibria depend on both the maturity structure and the level of government debt.[1] We contrast welfare in this model to an alternative setup in which monetary policy is optimal, lump-sum taxes may be adjusted to ensure the government's solvency condition never binds, but the distorting tax rate varies exogenously.

In the presence of distorting taxes, nominal rigidities, and long-term nominal government debt, we show that policy faces a fundamental conflict: stabilize inflation and allow output to fluctuate to accommodate fiscal needs versus stabilize output and use inflation to revalue debt to ensure government solvency. This conflict means that the first-best outcome—inflation and the output gap are perfectly smoothed and stabilized—is generally inconsistent with government solvency, so it is unattainable. The core of our analysis focuses on how the average maturity of government nominal bond affects the optimal equilibrium and the consequent distribution between inflation stabilization and output stabilization.

Two equations summarize equilibria under optimal monetary and fiscal policies. Let ρ index the average debt duration of the government's bond portfolio, where $\rho = 0$ makes all debt one-period and $\rho = 1$ makes all debt consols. Equilibrium k-step-ahead expectations of inflation, $\hat{\pi}$, and the output gap, $\hat{x}$, the two arguments of the government's loss function, are

$$E_t\hat{\pi}_{t+k} = \rho^k\hat{\pi}_t + \rho^k\alpha_\pi(L_t^b - L_{t-1}^b)$$
$$E_t\hat{x}_{t+k} = \rho^k\hat{x}_t + (1-\rho^k)\alpha_x L_t^b$$

where L_t^b is the Lagrange multiplier associated with government solvency and α_π and α_x are positive and functions of deep parameters.

These equilibrium expressions neatly encapsulate the policy problem. The first terms on the right stem from the welfare improvements that arise from smoothing. That both terms involve ρ^k means that longer maturity debt helps to smooth both inflation and output. The second terms bring in the government solvency dimension of optimal policy through the Lagrange multipliers. Now maturity has opposite effects on the two variables. As maturity extends, changes in the state of government solvency are permitted to affect future inflation more strongly, whereas the output gap becomes less responsive.

One-period debt, $\rho = 0$, underlies the work behind the consensus assignment. Optimal policy makes the price level a martingale—perfectly smoothes it—and forces the output gap to absorb disturbances. Following a disturbance at t, the price level is expected to remain unchanged (expected inflation is zero), while the output gap is expected to move to a new

[1] In a learning environment, Eusepi and Preston (2012) find that maturity structure and level of government debt have important consequences for stability.

level permanently. When nominal government bonds are perpetuities, optimal outcomes are starkly different. The output gap is a martingale and inflation adjusts permanently to exogenous shocks.

Intermediate maturities bring out the tradeoff between the smoothing and solvency aspects of optimal policy because both are operative. We explore this tradeoff in a variety of ways below. For any maturities short of perpetuities, $0 \leq \rho < 1$, as the forecast horizon extends, $k \to \infty$, expected inflation converges to zero whereas the expected output gap converges to $\alpha_x L_t^b$. In these cases, inflation is well anchored on zero, but the output gap's "anchor" varies with the state at t.

Key findings include: (1) there is always a role for current and future inflation innovations to revalue government debt, reducing reliance on distorting taxes; (2) the role of inflation in optimal fiscal financing increases with the average maturity of government debt; (3) as average maturity rises, it is optimal to tradeoff inflation for output stabilization; (4) inflation is relatively more important as a fiscal shock absorber in high-debt than in low-debt economies; (5) in some calibrations that are relevant to U.S. data, welfare under the fully optimal monetary and fiscal policies can be made equivalent to the welfare under the conventional optimal monetary policy with passively adjusting lump-sum taxes by extending the average maturity of bond.

Our analysis is built on two strands of literature. First, following the neoclassical literature on optimal taxation, when the government can only access to distortionary taxes, variations in tax rates generate dead-weight losses [Barro (1979)]. Maximization of welfare calls for smoothing tax rates and relying on the variations in real value of government debt to hedge against a fiscal shock. This is possible only when (i) the government can issue state-contingent bonds [Lucas and Stokey (1983), Chari et al. (1994)], or (ii) the government issues nominal bonds, but unexpected variations in inflation replicate state-contingent bonds [Bohn (1990) and Chari and Kehoe (1999)]. However, this work abstracts from monetary considerations by assuming flexible prices.

Another strand, the new Keynesian literature on optimal monetary policy, emphasizes that when prices are sticky, variation in aggregate price levels creates price dispersion that is an important source of welfare loss. A benevolent government minimizes price volatility. However, this strand tends to abstract from fiscal considerations by assuming non-distorting sources of revenue that maintain government solvency [Clarida et al. (1999a) and Woodford (2003)].

Our paper contributes to the literature by bringing the role of long-term nominal bond into the joint determination of monetary and fiscal policy. In particular, we are the first to consider how the long-term nominal bond affects optimal policy mix in a setting where price stability and fiscal financing smoothness are both in effect. We extend Sims (2013) which only considers *ad hoc* welfare functions and the extreme case of consol debt. This rich setting allows us to derive several novel findings. Specifically, we show that allowing adjustments in current & future price levels to revalue debt is generally part of an optimal policy and the role of inflation as a fiscal cushion increases with average maturity of debt. This finding has practical implications given the current fiscal and monetary situation.

We also connect to studies that focus on long-term bonds. Angeletos (2002) and Buera and Nicolini (2004) examine the optimal maturity structure of public debt to find that state-contingent debt can be constructed by non-contingent debt with different maturities. However, they consider only the case where prices are perfectly flexible and the government issues real debt. Therefore, they abstract from the monetary considerations. Woodford (1998), Cochrane (2001) and Sims (2001, 2013) study nominal government debt to argue that when outstanding government debt has long maturity, the government could finance higher government spending with a little bit of inflation spread over the maturity of the debt, effectively converting nominal debt into state-contingent real debt, as in Lucas and Stokey (1983). Both Cochrane and Sims employ *ad hoc* welfare functions to illustrate their points, so neither argues that revaluation of debt through inflation is a feature of a fully optimal policy. More importantly, they both consider a constant or exogenous real interest rate, downplaying the effects on real allocations of monetary and fiscal policies. Our contribution is to shed further light on the effect of long-term bond, first, by showing that as average maturity rises, it is optimal to tradeoff inflation for output stabilization; second, by showing the role of long-term bond in output smoothing and third, by studying the welfare equivalence between joint optimal policies and the conventional optimal monetary policy with passively adjusting lump-sum taxes.

The paper is organized as follows. Section 2 introduces the model. Section 3 develops the purely quadratic loss function and the linear constraints that prevent simultaneous stabilization of inflation and output. Section 4 studies optimal equilibrium under flexible price as a baseline. Section 5 characterizes the optimal equilibrium condition and joint optimal policy mix under sticky price. Section 6 outlines our calibration. Section 7 discusses three channels through which the average maturity of government debt affects optimal allocation. Section 8 studies how long-term bond affects the trade-off between inflation and output gap and the welfare. Section 9 studies the effect of different debt levels. Section 10 contrasts our result with the conventional optimal monetary policy. Section 11 concludes.

2 Model

We employ a standard new Keynesian economy that consists of a representative household with an infinite planning horizon, a collection of monopolistically competitive firms that produce differentiated goods, and a government. A fiscal authority finances exogenous expenditures with distorting taxes and debt and a monetary authority sets the short-term nominal interest rate.

2.1 Households

The economy is populated by a continuum of identical households. Each household has preferences defined over consumption, C_t, and hours worked, N_{jt}. Preferences are

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_{jt}) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{N_{jt}^{1+\varphi}}{1+\varphi} dj \right]$$

where σ^{-1} parameterizes the intertemporal elasticity of substitution, and φ^{-1} parametrizes the Frisch elasticity of labor supply.

Consumption is a CES aggregator defined over a basket of goods of measure one and indexed by j

$$C_t = \left[\int_0^1 C_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

where C_{jt} represents the quantity of good j consumed by the household in period t. The parameter $\epsilon > 1$ denotes the intratemporal elasticity of substitution across different varieties of consumption goods.[2] Each good j is produced using a type of labor that is specific to that industry, and N_{jt} denotes the quantity of labor supply of type j in period t. The representative household supplies all types of labor.

The aggregate price index P_t is

$$P_t = \left[\int_0^1 P_{jt}^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$$

where P_{jt} is the nominal price of the final goods produced in industry j.

Households maximize expected utility subject to the budget constraint

$$C_t + Q_t^S \frac{B_t^S}{P_t} + Q_t^M \frac{B_t^M}{P_t} = \frac{B_{t-1}^S}{P_t} + (1 + \rho Q_t^M) \frac{B_{t-1}^M}{P_t} + \int_0^1 \left(\frac{W_{jt}}{P_t} N_{jt} + \Pi_{jt} \right) dj + Z_t$$

[2] When $\epsilon \to \infty$, goods become perfect substitutes; when $\epsilon \to 1$, goods are neither substitutes nor complements: an increase in the price of one good has no effect on demand for other goods.

where W_{jt} is the nominal wage rate in industry j, Π_{jt} is the share of profits paid by the jth industry to the households, and Z_t is lump-sum government transfer payments. B_t^S is a one-period government bond with nominal price Q_t^S; B_t^M is a long-term government bond portfolio with price Q_t^M. The long-term bond portfolio is defined as perpetuities with coupons that decay exponentially, as in Woodford (2001). A bond issued at date t pays ρ^{k-1} dollars at date $t+k$, for $k \geq 1$ and $\rho \in [0,1]$ is the coupon decay factor that parameterizes the *average maturity* of the bond portfolio. A consol is the special case when $\rho = 1$ and one-period bonds arise when $\rho = 0$. The duration of the long-term bond portfolio B_t^M is $(1-\beta\rho)^{-1}$.

Household optimization yields the first-order conditions

$$\frac{W_{jt}}{P_t} = -\mu_t^W \frac{U_{n_j,t}}{U_{c,t}} \tag{1}$$

$$Q_t^S = \beta E_t \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \tag{2}$$

$$Q_t^M = \beta E_t \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} (1 + \rho Q_{t+1}^M) \tag{3}$$

where μ_t^W is an exogenous wage markup factor.[3] Combining (2) and (3) yields the no-arbitrage condition between one-period and long-term bonds

$$Q_t^M = E_t Q_t^S (1 + \rho Q_{t+1}^M) \tag{4}$$

2.2 Firms A continuum of monopolistically competitive firms produce differentiated goods. Production of good j is given by

$$Y_{jt} = A_t N_{jt}$$

where A_t is an exogenous aggregate technology shock, common across firms. Firm j faces the demand schedule

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon} Y_t$$

With demand imperfectly price-elastic, each firm has some market power, leading to the monopolistic competition distortion in the economy.

Another distortion stems from nominal rigidities. Prices are staggered, as in Calvo (1983),

[3] We follow Benigno and Woodford (2007) to include the time-varying exogenous wage markup in order to include a "pure" cost-push effect.

with a fraction $1-\theta$ of firms permitted to choose a new price, P_t^*, each period, while the remaining firms cannot adjust their prices. This pricing behavior implies the aggregate price index

$$P_t = [(1-\theta)(P_t^*)^{1-\epsilon} + \theta(P_{t-1})^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{5}$$

Firms that can reset their price choose P_t^* to maximize the expected sum of discounted future profits by solving

$$\max E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}[(1-\tau_{t+k})P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})]$$

subject to the demand schedule

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

where $Q_{t,t+k}$ is the stochastic discount factor for the price at t of one unit of composite consumption goods at $t+k$, defined by $Q_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}}$. Sales revenues are taxed at rate τ_t, Ψ_t is cost function, and $Y_{t+k|t}$ is output in period $t+k$ for a firm that last reset its price in period t.

The first-order condition for this maximization problem implies that the newly chosen price in period t, P_t^*, satisfies

$$\begin{aligned}\left(\frac{P_t^*}{P_t}\right)^{1+\epsilon\varphi} &= \frac{\epsilon}{\epsilon-1} \frac{E_t \sum_{k=0}^{\infty} (\beta\theta)^k \mu_{t+k}^W (\frac{Y_{t+k}}{A_{t+k}})^{\varphi+1} (\frac{P_{t+k}}{P_t})^{\epsilon(1+\varphi)}}{E_t \sum_{i=0}^{\infty} (\beta\theta)^k (1-\tau_{t+k}) U_{c,t+k} Y_{t+k} (\frac{P_{t+k}}{P_t})^{\epsilon-1}} \\ &= \frac{\epsilon}{\epsilon-1} \frac{K_t}{J_t}\end{aligned} \tag{6}$$

where K_t and J_t are aggregate variables that satisfy the recursive relations

$$K_t = \mu_t^W \left(\frac{Y_t}{A_t}\right)^{\varphi+1} + \beta\theta E_t K_{t+1} \pi_{t+1}^{\epsilon(1+\varphi)} \tag{7}$$

$$J_t = (1-\tau_t) U_{c,t} Y_t + \beta\theta E_t J_{t+1} \pi_{t+1}^{\epsilon-1} \tag{8}$$

2.3 Government The government consists of a monetary and a fiscal authority who face the consolidated budget constraint, expressed in real terms

$$\frac{(1+\rho Q_t^M) B_{t-1}^M}{P_t} = \frac{Q_t^M B_t^M}{P_t} + S_t \tag{9}$$

where S_t is the real primary budget surplus defined as

$$S_t = \tau_t Y_t - Z_t - G_t \tag{10}$$

G_t is government demand for the composite goods and Z_t is government transfer payments. We consider a fiscal regime in which both G_t and Z_t are exogenous processes and only τ_t adjusts endogenously to ensure government solvency. This assumption breaks Ricardian equivalence, so the government's budget and the dynamics of public debt matter for welfare and monetary policy can have important fiscal consequences.

An intertemporal equilibrium—or solvency—condition links the real market value of outstanding government bonds to the expected present value of primary surpluses[4]

$$(1+\rho Q_t^M)\frac{B_{t-1}^M}{P_t} = E_t \sum_{k=0}^{\infty} R_{t,t+k} S_{t+k} \tag{11}$$

where $R_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}}$ is the k-period real discount factor.

The left-hand side of (11) highlights a key role of long-term bonds. With only one-period bonds, $\rho = 0$, the nominal value of outstanding government bonds, B_{t-1}^M, is predetermined, so an unexpected change to the present value of primary surpluses must be absorbed entirely by surprise inflation or deflation at time t. Long-term bonds, $\rho > 0$, imply that the nominal value of government bond, $(1+\rho Q_t^M)B_{t-1}^M$, is no longer predetermined. Because the nominal bond price Q_t^M, depends on expected future riskless short-term nominal interest rates[5]

$$Q_t^M = E_t \sum_{k=0}^{\infty} \frac{\rho^k}{i_t i_{t+1} \dots i_{t+k}} \tag{12}$$

solvency condition (11) may be written as

$$\underbrace{\left[1 + E_t \sum_{k=0}^{\infty} \frac{\rho^k}{i_t i_{t+1} \dots i_{t+k}}\right]}_{\text{current and future monetary policy}} \frac{B_{t-1}^M}{P_t} = \underbrace{E_t \sum_{k=0}^{\infty} R_{t,t+k} S_{t+k}}_{\text{current and future fiscal policy}} \tag{13}$$

Now an unexpected change to the present value of primary surpluses could be absorbed by adjustments in current and future nominal interest rates, reducing the reliance on current inflation.

[4]See Appendix A for the derivation of this condition.

[5]The riskless short-term nominal gross interest rate is defined by $i_t = \left[Q_t^S\right]^{-1}$. See Appendix A for the derivation of condition (12).

Equilibrium condition (13) reflects a fundamental symmetry between monetary and fiscal policies. The price level today must be consistent with expected future monetary and fiscal policies, whether those policies are set optimally or not. Bond maturity matters: so long as the average maturity exceeds one period, $\rho > 0$, expected future monetary policy in the form of choices of the short-term nominal interest rate, i_{t+k}, plays a role in determining the current price level.

2.4 Equilibrium

Market clearing in the goods market requires

$$Y_t = C_t + G_t \tag{14}$$

and market clearing in labor market requires

$$\Delta_t^{\frac{1}{1+\varphi}} Y_t = A_t N_t \tag{15}$$

where $\Delta_t = \int_0^1 (\frac{P_{jt}}{P_t})^{-\epsilon(1+\varphi)} dj$ denotes the the measure of price dispersion across firms and satisfies the recursive relation

$$\Delta_t = (1-\theta)\left[\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}\right]^{\frac{\epsilon(1+\varphi)}{\epsilon-1}} + \theta\pi_t^{\epsilon(1+\varphi)}\Delta_{t-1} \tag{16}$$

Price dispersion is the source of welfare losses from inflation variability.

3 Fully Optimal Policy

In the fully optimal policy problem, government chooses functions for the tax rate, τ_t, and the short-term nominal interest rate, i_t, taking exogenous processes for technology, A_t, the wage markup, μ_t^W, government purchases, G_t, and transfers, Z_t, as given. We derive how the optimal policy and welfare vary with the average maturity of government debt, as indexed by ρ. We consider the case of a steady state distorted by distortionary tax and monopolistic competition and focus on optimal policy commitment, adopting Woodford's (2003) "timeless perspective."

3.1 Linear-Quadratic Approximation

We compute a linear-quadratic approximation to the nonlinear optimal solutions, using the methods that Benigno and Woodford (2004) develop. This allows us to characterize the optimal policy responses to fluctuations in the exogenous disturbance processes within a neighborhood of the steady state.

In this model, distorting taxes and monopolistic competition conspire to make the deterministic steady state inefficient, so an *ad hoc* linear-quadratic representation of the problem does not yield an accurate approximation of the optimal policy.[6] Benigno and Woodford (2004) show that a correct linear-quadratic approximation is still possible by properly utilizing information from micro-foundations. Their approach computes a second-order approximation to the model's structural equations and uses an appropriate linear combination of those equations to eliminate the linear terms in the second-order approximation to the welfare measure to obtain a purely quadratic expression.

We follow Benigno and Woodford's micro-founded linear-quadratic approach for three reasons. First, it allows us to obtain neat analytical solutions that help us to characterize the properties of optimal policies and separate out the channels through which long-term bonds affect optimal allocation. Second, the framework nests conventional analyses of both optimal inflation-smoothing and optimal tax-smoothing, providing an integrated approach to the two literatures. Third, the quadratic welfare criterion is independent of policy, which permits us to compare our results to alternative sub-optimal policies.

Welfare losses experienced by the representative household are, up to a second-order approximation, proportional to[7]

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(q_\pi\hat{\pi}_t^2+q_x\hat{x}_t^2\right) \tag{17}$$

where the relative weight on output stabilization depends on model parameters

$$\frac{q_x}{q_\pi}\equiv\frac{\kappa}{\epsilon}\left[1+\frac{s_c^{-1}\sigma}{\varphi+s_c^{-1}\sigma}\frac{(1+w_g)(1+w_\tau)-s_c^{-1}(1+w_g+w_\tau)}{(\Phi^{-1}-1)\Gamma+(1+w_g)(1+\varphi)}\right]$$

$\hat{x}_t$ denotes the welfare-relevant output gap, defined as the deviation between $\hat{Y}_t$ and its efficient level $\hat{Y}_t^e$, $\hat{x}_t\equiv\hat{Y}_t-\hat{Y}_t^e$. Efficient output, $\hat{Y}_t^e$, depends on the four fundamental shocks and is given by $\hat{Y}_t^e=q_A\hat{A}_t+q_G\hat{G}_t+q_Z\hat{Z}_t+q_W\mu_t^W$.[8] $w_g=(\bar{Z}+\bar{G})/\bar{S}$ is steady-state government outlays to surplus ratio, $w_\tau=\bar{\tau}/1-\bar{\tau}$, $s_c=\bar{C}/\bar{Y}$ is the steady-state consumption

[6] One convenient way to eliminate the inefficiency of steady state is to assume an employment subsidy that offsets the distortion due to the market power of monopolistically-competitive price-setters or distorting tax, so that the steady state with zero inflation involves an efficient level of output. We instead consider a more realistic case, where such employment subsidy is not available. See Kim and Kim (2003) and Woodford (2011) for more discussions.

[7] See Appendices C–F for detailed derivations.

[8] Parameters q_A, q_G, q_Z and q_W are defined in appendix F.

to GDP ratio, and

$$\begin{aligned}
\kappa &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{s_c^{-1}\sigma+\varphi}{1+\epsilon\varphi} \\
\Gamma &= (s_c^{-1}\sigma+\varphi)(1+w_g)+s_c^{-1}\sigma w_\tau - w_\tau(1+w_g) \\
\Phi &= 1-(1-\bar{\tau})\frac{\epsilon-1}{\epsilon}
\end{aligned}$$

Note that $-\frac{U_n}{U_c}=(1-\Phi)MPN$, so Φ, which measures the inefficiency of the steady state, depends on the steady state tax rate, $\bar{\tau}$, and the elasticity of substitution between differentiated goods, ϵ.

3.2 Linear Constraints

Constraints on the optimization problem come from log-linear approximations to the model equations. The first constraint comes from the aggregate supply relation between current inflation and the output gap

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa(\hat{x}_t + \psi\hat{\tau}_t) + u_t \tag{18}$$

where u_t is a composite cost-push shock that depends on the four exogenous disturbances

$$u_t \equiv \underbrace{\kappa\left[q_A - \frac{1+\varphi}{\varphi+\sigma s_c^{-1}}\right]}_{u_A}\hat{A}_t + \underbrace{\kappa\left[q_G - \frac{\sigma}{\varphi+\sigma s_c^{-1}}\frac{s_g}{s_c}\right]}_{u_G}\hat{G}_t + \underbrace{\kappa q_Z}_{u_Z}\hat{Z}_t + \underbrace{\kappa\left[q_W + \frac{1}{\varphi+\sigma s_c^{-1}}\right]}_{u_W}\hat{\mu}_t^W \tag{19}$$

The exogenous disturbances generate cost-push effects through (19) because with a distorted steady state, they generate a time-varying gap between the flexible-price equilibrium level of output and the efficient level of output. If the steady state were not distorted, only variations in wage markups would have cost-push effects. This is why wage markups are regarded as "pure" cost-push disturbances.[9]

When $\hat{\tau}_t$ is exogenous, $\kappa\psi\hat{\tau}_t + u_t$ prevents complete stabilization of inflation and the welfare-relevant output gap. Iterating forward on (18) yields

$$\hat{\pi}_t = E_t\sum_{k=0}^{\infty}\beta^k\kappa\hat{x}_{t+k} + U_t$$

where $U_t \equiv E_t\sum_{k=0}^{\infty}\beta^k(\kappa\psi\hat{\tau}_{t+k}+u_{t+k})$ determines the degree to which stabilization of inflation and output gap is not possible. This is the only source of trade-off between stabilization

[9] See Benigno and Woodford (2004) for detailed discussions.

of inflation and output gap in conventional new Keynesian optimal monetary policy analyses [for example, Galí (1991)].

When $\hat{\tau}_t$ is chosen optimally along with monetary policy, then $\hat{\tau}_t$ can be set to fully absorb cost-push shocks, making simultaneous stabilization of inflation and the output gap possible. Benigno and Woodford (2004) rewrite (18) as

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa \hat{x}_t + \kappa\psi(\hat{\tau}_t - \hat{\tau}_t^*) \tag{20}$$

where $\hat{\tau}_t^* \equiv -\frac{1}{\kappa\psi} u_t$ is the tax rate that offsets the cost-push shock. Expression (20) describes the trade-off relation between inflation and output that fiscal policy faces because tax rates can help stabilize output and inflation by offsetting variations in cost-push distortions.

A second constraint arises from the household's Euler equation. After imposing market clearing it may be written as

$$\hat{x}_t = E_t[\hat{x}_{t+1}] - \frac{s_c}{\sigma}\left(\hat{i}_t - E_t[\hat{\pi}_{t+1}]\right) + v_t \tag{21}$$

where the composite aggregate demand shock, v_t, is

$$v_t \equiv \underbrace{q_A(\rho_A - 1)}_{v_A}\hat{A}_t + \underbrace{(q_G - s_g)(\rho_G - 1)}_{v_G}\hat{G}_t + \underbrace{q_Z(\rho_Z - 1)}_{v_Z}\hat{Z}_t + \underbrace{q_W(\rho_W - 1)}_{v_W}\hat{\mu}_t^W \tag{22}$$

Alternatively, (21) can be written as

$$\hat{x}_t = E_t[\hat{x}_{t+1}] + \frac{s_c}{\sigma}E_t[\hat{\pi}_{t+1}] - \frac{s_c}{\sigma}\left(\hat{i}_t - \hat{i}_t^*\right) \tag{23}$$

where $\hat{i}_t^* \equiv \frac{\sigma}{s_c} v_t$ is the setting of the short-term nominal interest rate that exactly offsets the composite demand-side shock.[10] Expression (23) makes clear how monetary policy can offset variations in demand-side distortions.

If (20) and (23) were the only constraints facing policy makers, it would be possible to choose monetary and tax policies to completely stabilize inflation and output. Policy could achieve the first-best outcome, $\hat{\pi}_t = \hat{x}_t = 0$, by setting

$$\hat{\tau}_t = \hat{\tau}_t^* \qquad \hat{i}_t = \hat{i}_t^* \tag{24}$$

In the absence of any additional constraints on the policy problem, policy authorities who are free to choose paths for the short-term nominal interest rate and tax rate can achieve

[10] Note that $\hat{i}_t^* = \frac{\sigma}{s_c} E_t[(\hat{y}_{t+1}^e - \hat{y}_t^e) - s_g(\hat{G}_{t+1} - \hat{G}_t)]$, giving it an interpretation as the efficient level of the real interest rate.

the unconstrained maximum of welfare. To achieve this first-best outcome, policy must have access to a non-distorting source of revenues or state-contingent debt that can adjust to ensure that the government's solvency requirements do not impose additional restrictions on achievable outcomes.

When non-distorting revenues are not available, the government can convert nominal bonds into state-dependent real bonds. If the government issues nominal bonds with average maturity indexed by ρ, fiscal solvency implies the additional constraint

$$\hat{b}^M_{t-1} + f_t = \beta \hat{b}^M_t + (1-\beta)\frac{\bar{\tau}}{s_d}(\hat{\tau}_t + \hat{x}_t) + \hat{\pi}_t + \beta(1-\rho)\hat{Q}^M_t \tag{25}$$

where $s_d \equiv \bar{S}/\bar{Y}$ is the steady-state surplus to output ratio and f_t is a composite fiscal shock that reflects all four exogenous disturbances to the government's flow constraint

$$f_t \equiv \underbrace{-(1-\beta)\frac{\bar{\tau}}{s_d}q_A}_{f_A}\hat{A}_t + \underbrace{(1-\beta)\left(\frac{s_g}{s_d} - \frac{\bar{\tau}}{s_d}q_G\right)}_{f_G}\hat{G}_t + \underbrace{(1-\beta)\left(\frac{s_z}{s_d} - \frac{\bar{\tau}}{s_d}q_Z\right)}_{f_Z}\hat{Z}_t \underbrace{-(1-\beta)\frac{\bar{\tau}}{s_d}q_W}_{f_W}\hat{\mu}^W_t \tag{26}$$

In general, all disturbances have fiscal consequences through (25) and (26), because nondistorting taxes are not available to offset their impacts on the government's budget.

Absence of arbitrage between short-term and long-term bonds delivers the fourth constraint on the optimal policy program

$$\beta\rho E_t \hat{Q}^M_{t+1} = \hat{Q}^M_t + \hat{i}_t \tag{27}$$

Iterating on (27) and applying a terminal condition yields

$$\hat{Q}^M_t = -E_t \sum_{k=0}^{\infty} (\beta\rho)^k \hat{i}_{t+k} \tag{28}$$

Defining the long-term interest rate i^M_t as the yield to maturity, $i^M_t \equiv \frac{1}{Q^M_t} - (1-\rho)$, we obtain the term structure of interest rates

$$\hat{i}^M_t = \frac{1-\beta\rho}{1-\beta} E_t \sum_{k=0}^{\infty} (\beta\rho)^k \hat{i}_{t+k} \tag{29}$$

When $\rho = 0$, all bonds are one period, $\hat{i}^M_t = \frac{1}{1-\beta}\hat{i}_t$, the long-term interest rate at time t is proportional to the current short-term interest rate, so any disturbance to the long rate will also affect the current short rate. When $\rho > 0$, the long-term interest rate at time t

is determined by the whole path of future short-term interest rates, making intertemporal smoothing possible. A disturbance to the long-term interest rate can be absorbed by adjusting future short-term interest rates, with no change in the current short rate. By separating current and future monetary policies, long bonds provide policy additional leverage.

Iterating forward on the government's budget constraint (25) and imposing transversality and the no-arbitrage condition (47), we obtain the intertemporal equilibrium condition

$$\underbrace{\hat{b}^M_{t-1} + F_t}_{\text{fiscal stress}} = \hat{\pi}_t + \frac{\sigma}{s_c}\hat{x}_t + (1-\beta)E_t\sum_{k=0}^{\infty}\beta^k[b_\tau(\hat{\tau}_{t+k} - \hat{\tau}^*_{t+k}) + b_x\hat{x}_{t+k}] + \underbrace{E_t\sum_{k=0}^{\infty}(\beta\rho)^{k+1}(\hat{i}_{t+k} - \hat{i}^*_{t+k})}_{\text{due to long-term bonds}} \tag{30}$$

where $b_\tau = \frac{\bar{\tau}}{s_d}$, $b_x = \frac{\bar{\tau}}{s_d} - \frac{\sigma}{s_c}$ and

$$F_t = E_t\sum_{k=0}^{\infty}\beta^k f_{t+k} - (1-\beta)\frac{\bar{\tau}}{s_d}E_t\sum_{k=0}^{\infty}\beta^k\hat{\tau}^*_{t+k} + E_t\sum_{k=0}^{\infty}[\beta^{k+1} - (\beta\rho)^{k+1}]\hat{i}^*_{t+k} \tag{31}$$

The sum $\hat{b}^M_{t-1} + F_t$ summarizes the fiscal stress that prevents complete stabilization of inflation and the welfare-relevant output gap. Given the definitions of τ^* and i^*, F_t reflects fiscal stress stemming from three conceptually distinct but related sources: the composite fiscal shock, f_t, the composite cost-push shock, u_t (through τ^*_t), and the composite aggregate demand shock, v_t (through i^*_t).[11]

Contrasting (30) to the one-period bond case in Benigno and Woodford (2004), the presence of long-term bonds gives a role to expectations of future monetary policies. Monetary and fiscal policy can be coordinated so that households' expectations about future policies affect long-term interest rates to offset part of the overall fiscal stress in the economy.

With F_t fluctuating exogenously, complete stabilization of inflation and output, $\hat{\pi}_t = \hat{x}_t = 0$, which implies $\hat{\tau}_t = \hat{\tau}^*_t, \hat{i}_t = \hat{i}^*_t$, will not generally satisfy (30) and the government would be insolvent. The additional fiscal solvency constraint prevents the first-best allocation from being achievable. Any feasible allocation involves a tension between stabilization of inflation and output gap, so the optimal policy must balance this tension.

[11] F_t corresponds to the fiscal stress that Benigno and Woodford (2004) define, but here it is extended to the case of long-term bonds.

4 Optimal Policy Analytics: Flexible Prices

In this section we characterize optimal equilibrium and policy assignment for the special case of completely flexible prices. This case serves as a baseline, since with flexible prices the trade-off between inflation and output gap disappears. It also connects to earlier work by Chari et al. (1996) and Chari and Kehoe (1999), except that they considered only real government bonds, while we consider nominal bonds. Flexible prices emerge when $\theta = 0$, which implies $\kappa = \infty$ and $q_\pi = 0$. Costless inflation converts the loss function from (17) to

$$\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t q_x \hat{x}_t^2 \tag{32}$$

and the optimal policy problem minimizes (32) subject to the sequence of constraints

$$\hat{x}_t + \psi(\hat{\tau}_t - \hat{\tau}_t^*) = 0 \tag{33}$$

$$\hat{x}_t + \frac{s_c}{\sigma}(\hat{i}_t - \hat{i}_t^*) - E_t[\hat{x}_{t+1}] - \frac{s_c}{\sigma} E_t[\hat{\pi}_{t+1}] = 0 \tag{34}$$

$$\begin{aligned} \hat{b}_{t-1}^M + F_t = \hat{\pi}_t + & \frac{\sigma}{s_c}\hat{x}_t + (1-\beta) E_t \sum_{k=0}^{\infty} \beta^k [b_\tau(\hat{\tau}_{t+k} - \hat{\tau}_{t+k}^*) + b_x \hat{x}_{t+k}] \\ & + E_t \sum_{k=0}^{\infty} (\beta\rho)^{k+1} (\hat{i}_{t+k} - \hat{i}_{t+k}^*) \end{aligned} \tag{35}$$

The optimal solution entails $\hat{x}_t = 0$ at all times, which can be achieved if fiscal policy follows $\hat{\tau}_t = \hat{\tau}_t^*$ and monetary policy sets the short-term real interest rate as $\hat{i}_t - E_t \hat{\pi}_{t+1} = \hat{i}_t^*$. In this optimal policy assignment, fiscal policy stabilizes the output gap, monetary policy stabilizes expected inflation and the maturity structure of debt determines the timing of inflation. Equilibrium inflation satisfies

$$\hat{b}_{t-1}^M + F_t = \hat{\pi}_t + E_t \sum_{k=1}^{\infty} (\beta\rho)^k \hat{\pi}_{t+k} \tag{36}$$

so increases in factors that prevent complete stabilization of the objectives $\hat{b}_{t-1}^M + F_t$, raise the expected present value of inflation. When $\rho > 0$, (36) implies that long-term bonds allow the government to trade off inflation today for inflation in the future. The longer the average maturity, the farther into the future inflation can be postponed. This conclusion is reminiscent of Cochrane's (2001) optimal inflation-smoothing result.

When $\rho = 0$ and all bonds are one-period, (36) collapses to

$$\hat{b}^M_{t-1} + F_t = \hat{\pi}_t \tag{37}$$

and, as Benigno and Woodford (2007) emphasize, "optimal policy will involve highly volatile inflation and extreme sensitivity of inflation to fiscal shocks."

Flexible prices neglect the welfare costs of inflation. When prices are sticky and inflation volatility is costly, the optimal allocation should balance variations in inflation against variations in the output gap.

5 Optimal Policy Analytics: Sticky Prices

In the case where prices are sticky, the optimization problem finds paths for $\{\hat{\pi}_t, \hat{x}_t, \hat{\tau}_t, \hat{i}_t, \hat{b}^M_t, \hat{Q}^M_t\}$ that minimize

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t[\hat{\pi}_t^2 + \lambda\hat{x}_t^2], \quad \lambda \equiv \frac{q_x}{q_\pi} \tag{38}$$

subject to the sequence of constraints

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa\hat{x}_t + \kappa\psi(\hat{\tau}_t - \hat{\tau}^*_t) \tag{39}$$

$$\hat{x}_t = E_t[\hat{x}_{t+1}] + \frac{s_c}{\sigma}E_t[\hat{\pi}_{t+1}] - \frac{s_c}{\sigma}(\hat{i}_t - \hat{i}^*_t) \tag{40}$$

$$\hat{b}^M_{t-1} = \beta\hat{b}^M_t + (1-\beta)\frac{\bar{\tau}}{s_d}(\hat{\tau}_t + \hat{x}_t) + \hat{\pi}_t + \beta(1-\rho)\hat{Q}^M_t - f_t \tag{41}$$

$$\hat{Q}^M_t = \beta\rho E_t\hat{Q}^M_{t+1} - \hat{i}_t \tag{42}$$

Taking first-order conditions with respect to $\hat{\pi}_t, \hat{x}_t, \hat{i}_t, \hat{\tau}_t, \hat{b}^M_t$ and $\hat{Q}^M_t$, we obtain the following optimality conditions:

$$\hat{\pi}_t = -\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d}(L^b_t - L^b_{t-1}) - L^b_t + \frac{1}{\beta}L^q_{t-1} \tag{43}$$

$$\lambda\hat{x}_t = (\psi^{-1} - 1)(1-\beta)\frac{\bar{\tau}}{s_d}L^b_t - \frac{\sigma}{s_c}L^q_t + \frac{\sigma}{s_c}\frac{1}{\beta}L^q_{t-1} \tag{44}$$

$$\beta(1-\rho)L^b_t - L^q_t + \rho L^q_{t-1} = 0 \tag{45}$$

$$E_tL^b_{t+1} - L^b_t = 0 \tag{46}$$

where L^b_t and L^q_t are Lagrange multipliers corresponding to (41) and (42).The Lagrange multiplier to (39) is proportional to L^b_t and the Lagrange multiplier to (40) is proportional to L^q_t. This allows us to substitue them out and express the optimal inflation and output gap merely by L_t^b and L_t^q. We solve (39)–(46) for state-contingent paths of $\{\hat{\pi}_t, \hat{x}_t, \hat{i}_t, \hat{\tau}_t, \hat{b}_t^M, \hat{Q}_t^M, L_t^q, L_t^b\}$.

L_t^b measures the shadow cost of government budget resources. Note from (41), the only control variable that appear to adjust the government budget resources is tax rate τ_t. Therefore, we could think of L_t^b as a measurement of how binding the fiscal solvency constraint is on the conduct of fiscal policy. L_t^b follows a martingale according to (46), implying intertemporal smoothing in fiscal financing. The extent of such smoothing, however, depends on the property of bonds market. Usually with nominal bond, the bond price $\hat{Q}_t^M$ could behave as a nominal buffer to facilitate fiscal financing. $\hat{Q}_t^M$ is determined by the whole path of current and future short-term interest rates, by iterating (42),

$$\hat{Q}_t^M = -\hat{i}_t - E_t \sum_{k=1}^{\infty} (\beta\rho)^k \hat{i}_{t+k} \tag{47}$$

Therefore, by allowing $\hat{Q}_t^M$ to hedge against fiscal shock, the fiscal solvency condition imposes constraint on current and future monetary policies. How much stress is transmitted to current interest rate $\hat{i}_t$ and how much stress is transmitted to future interest rates $\hat{i}_{t+k}$ is determined by the average maturity ρ. If $\rho = 0$, $\hat{Q}_t^M = -\hat{i}_t$, the fiscal hedging stress on $\hat{Q}_t^M$ is fully transmitted to current interest rate, therefore current monetary policy is most constrained by fiscal solvency. If $\rho > 0$, $\hat{Q}_t^M$ also depends on future short-term interest rates. The fiscal hedging stress on $\hat{Q}_t^M$ can be absorbed by future short-term interest rates, reducing changes in the current short rate, in this way, current monetary policy is much less constrained. Therefore, L_t^q measures how tightly the current interest rate $\hat{i}_t$ is constrained by fiscal solvency condition. L_t^q is determined by the the entire history of L_{t-k}^b, and the degree of history dependence rises with the average maturity of government debt,

$$L_t^q = \beta(1-\rho) \sum_{k=0}^{\infty} \rho^k L_{t-k}^b \tag{48}$$

If $\rho = 0$, $L_t^q = \beta L_t^b$, eliminating the history dependence. This is because $\hat{Q}_t^M = -\hat{i}_t$, if nominal asset is used as fiscal hedging, then it must be the case that the disturbance at time t is fully absorbed by interest rate at time t, no intertemporal smoothing is available. Monetary policy is almost as constrained by fiscal solvency as fiscal policy itself is.[12] In the opposite extreme, consols make $\rho = 1$, so $L_t^q = 0$. Current monetary policy is not constrained, regardless of how binding the government's budget has been in the past, as long as future monetary policies are expected to adjust appropriately.[13] From this perspective,

[12] This is precisely the exercise that finds the combination of active monetary/passive fiscal policies yields highest welfare [Schmitt-Grohé and Uribe (2007) and Kirsanova and Wren-Lewis (2012)].

[13] Sims (2013) limits attention to this case.

the role of long-term bond is to relax current monetary policy from fiscal stress by involving more commitment to future monetary policies. Debt maturity introduces a fresh role for expected monetary policy choices by allowing those expectations to help ensure government solvency.[14]

We examine some special cases that allow us to characterize the optimal equilibrium and the consequent stabilization role of fiscal and monetary policy analytically.

5.1 Only One-Period Bonds

Suppose the government issues only one-period bonds, rolled over every period. Then $\rho = 0$ and (48) and (29) reduce to

$$L_t^q = \beta L_t^b \tag{49}$$

$$\hat{i}_t^M = \frac{1}{1-\beta}\hat{i}_t \tag{50}$$

Long-term and short-term interest rates are identical, so L_t^b and L_t^q covary perfectly. In this case, the expressions for inflation, (43), and the output gap, (44), become

$$\hat{\pi}_t = -\left(\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d} + 1\right)\left(L_t^b - L_{t-1}^b\right) \tag{51}$$

$$\lambda\hat{x}_t = \left[\left(\psi^{-1} - 1\right)(1-\beta)\frac{\bar{\tau}}{s_d} - \beta\frac{\sigma}{s_c}\right] L_t^b + \frac{\sigma}{s_c} L_{t-1}^b \tag{52}$$

Condition (51) implies that inflation is proportional to the forecast error in L_t^b.[15] Because (46) requires there are no forecastable variations in L_t^b, the expectation of inflation is zero, implying perfect smoothing of the price level

$$E_t\hat{\pi}_{t+1} = 0 \quad \Rightarrow \quad E_t\hat{p}_{t+1} = \hat{p}_t \tag{53}$$

Condition (52) makes the output gap a weighted average of L_t^b and L_{t-1}^b. Taking expectations yields

$$\lambda(E_t\hat{x}_{t+1} - x_t) = \frac{\sigma}{s_c}(L_t^b - L_{t-1}^b) \tag{54}$$

so the expected change in the output gap next period is proportional to the surprise in the multiplier on government solvency today. The optimal degree of output-gap smoothing varies with λ, the weight on output in the loss function. The bigger is λ, the more smoothing of the output gap. Flexible prices are a special case with $\lambda = \infty$ and perfect smoothing of the

[14] The new Keynesian literature emphasizes the role of expected monetary policy via its influence of the entire future path of *ex-ante* real interest rates that enter the Euler equation, (21). The role we are discussing for expected monetary policy is in addition to this conventional role.

[15] First-order condition (46) makes $E_t L_{t+1}^b = L_t^b$, so the surprise is $L_{t+1}^b - E_t L_{t+1}^b = \Delta L_{t+1}^b$.

output gap. Under most calibrations, λ is quite small, implying little smoothing of output. But the martingale property of L_t^b implies smoothing of expected future output gaps after a one-time jump. Taking expectations of (54) yields

$$\hat{x}_t \neq E_t\hat{x}_{t+1} = E_t\hat{x}_{t+2} = ... = E_t\hat{x}_{t+k} = ... \tag{55}$$

Taken together, (53) and (55) imply that with only one-period bonds, optimal policies smooth the price level, while using fluctuations in the output gap to absorb innovations in fiscal conditions. The reason is apparent: with no long-term bonds, policy cannot smooth inflation in the future and surprise inflation—and the resulting price dispersion—is far more costly that variations in the output gap; it is optimal to minimize inflation variability and use output as a shock absorber.

In this equilibrium, monetary and fiscal policies follow the rules

$$\hat{\tau}_t - \hat{\tau}_t^* = \frac{1}{\kappa\psi}(\hat{\pi}_t - \kappa\hat{x}_t) \tag{56}$$

$$\hat{i}_t - \hat{i}_t^* = -\frac{(\sigma/s_c)^2}{\lambda(\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d} + 1)}\hat{\pi}_t \tag{57}$$

so monetary policy pins down inflation by offsetting variations in demand-side disturbances and fiscal policy stabilizes the output gap by responding to monetary policy and cost-push disturbances.

5.2 Only Consols

Suppose the government issues only consols. With $\rho = 1$, (48) and (29) reduce to

$$L_t^q = L_{t-1}^q = 0 \tag{58}$$

$$\hat{i}_t^M = E_t\sum_{k=0}^{\infty}\beta^k\hat{i}_{t+k} \tag{59}$$

In the case of consols, the long-term interest rate is determined by the entire path of future short-term interest rates. Fiscal stress that moves long rates need not change short rates contemporaneous, so long as the expected path of short rates satisfies (59). Inflation and output are now

$$\hat{\pi}_t = -\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d}\left(L_t^b - L_{t-1}^b\right) - L_t^b \tag{60}$$

$$\lambda\hat{x}_t = \left(\psi^{-1} - 1\right)(1-\beta)\frac{\bar{\tau}}{s_d}L_t^b \tag{61}$$

Combining (60) and (61) yields

$$\hat{\pi}_t + \frac{\lambda}{\kappa(1-\psi)}(\hat{x}_t - \hat{x}_{t-1}) + \frac{\lambda}{(\psi^{-1}-1)(1-\beta)\frac{\bar{\tau}}{s_d}}\hat{x}_t = 0 \tag{62}$$

an expression that generalizes the "flexible target criterion" found in conventional optimal monetary policy exercises in new Keynesian models.[16]

Condition (61) makes the output gap proportional to L_t^b. The martingale property of L_t^b makes the output gap also a martingale, so the gap is perfectly smoothed

$$E_t\hat{x}_{t+1} = \hat{x}_t \tag{63}$$

Taking expectations of (60) and combining with (61), we have

$$E_t\hat{\pi}_{t+1} - \hat{\pi}_t = \frac{\lambda}{\kappa(1-\psi)}(\hat{x}_t - \hat{x}_{t-1}) \tag{64}$$

Condition (64) implies that the expected change in the inflation next period is proportional to the forecasting error of $\hat{x}_t$. The degree of inflation smoothing changes inversely with λ, the weight on output in the loss function, while the degree of inflation smoothing varies proportionally with κ, the slope of the Phillips curve.

Combining (63) and (64), we draw opposite conclusions about the assignment between inflation and output gap to the case of only one-period bonds. With only consols, intertemporal smoothing in the shadow price of the government budget constraint, L_t^b, smoothes the output gap, relying on fluctuations in inflation to absorb innovations in fiscal conditions. To understand this, refer to the government solvency condition

$$\hat{b}_{t-1}^M + \beta\rho\hat{Q}_t^M - \hat{\pi}_t = (1-\beta)E_t\sum_{k=0}^{\infty}\beta^k(\hat{r}_{t,t+k} + \hat{s}_{t+k}) \tag{65}$$

where $\hat{r}_{t,t+k}$ is the log-linearized real discount rate. Consols introduce the possibility that the bond price $\hat{Q}_t^M$ can behave as a fiscal shock absorber: bad news about future surpluses can reduce the value of outstanding bonds, leaving the real discount rate unaffected. A constant real discount rate smoothes the output gap, which explains the absence of forecastable variations in the output gap. Variations in the bond price $\hat{Q}_t^M$ correspond to adjustments in future inflation. The longer the duration of debt—higher ρ—the less is the required change

[16] Notice that as $\psi \to 0$, which occurs as the steady state distorting tax rate approaches 0, $L_t^b \to 0$ and (62) approaches the conventional flexible target criterion with lump-sum taxes $\hat{\pi}_t + \frac{\lambda}{\kappa}(\hat{x}_t - \hat{x}_{t-1}) = 0$ so that the optimal inflation rate should vary with both the the rate of change in the output gap and the level of the gap [see Woodford (2011) and references therein].

in bond prices and future inflation for a given change in the present-value of surpluses. Although with consols it is optimal to allow surprise inflation and deflation to absorb shocks, the expectation of inflation is stabilized after a one-time jump

$$\hat{\pi}_t \neq E_t\hat{\pi}_{t+1} = E_t\hat{\pi}_{t+2} = ... = E_t\hat{\pi}_{t+k} = ... \tag{66}$$

Optimal monetary and fiscal policy obey

$$\hat{\tau}_t - \hat{\tau}_t^* = \frac{1}{\kappa\psi}(\hat{\pi}_t - \beta E_t\hat{\pi}_{t+1} - \kappa\hat{x}_t) \tag{67}$$

$$\hat{i}_t - \hat{i}_t^* = E_t\hat{\pi}_{t+1} = -\frac{\lambda}{(1/\psi - 1)(1-\beta)\frac{\bar{\tau}}{s_d}}\hat{x}_t \tag{68}$$

Monetary policy pins down expected inflation, but not actual inflation. Expected inflation determines how much fiscal stress is absorbed through changes in long-term bond prices and with more adjustment occurring through inflation, the output gap is better stabilized. Fiscal policy determines inflation by responding to monetary policy and cost-push side disturbances.

5.3 General Case

We briefly consider intermediate value for the average duration of debt, $0 < \rho < 1$. Rewrite (43) and (44) using the lag-operator notation, $\mathbb{L}^j x_t \equiv x_{t-j}$

$$\hat{\pi}_t = -\frac{(1-\beta)\frac{\bar{\tau}}{s_d}}{\kappa\psi}(1-\mathbb{L})L_t^b - (1-\mathbb{L})(1-\rho\mathbb{L})^{-1}L_t^b \tag{69}$$

$$\lambda\hat{x}_t = (\psi^{-1} - 1)(1-\beta)\frac{\bar{\tau}}{s_d}L_t^b - \frac{\sigma\beta}{s_c}(1-\rho)(1-\beta^{-1}\mathbb{L})(1-\rho\mathbb{L})^{-1}L_t^b \tag{70}$$

The optimality condition for debt that requires L_t^b to be a martingale may be written as

$$(1-\mathbb{B})E_{t-1}L_t^b = 0 \tag{71}$$

where $\mathbb{B}$ is the backshift operator, defined as $\mathbb{B}^{-j}E_t\xi_t \equiv E_t\xi_{t+j}$.

Taking expectations of (69) and (70), and applying (71), we obtain a general relation

$$E_t\hat{\pi}_{t+k} = \underbrace{\rho^k\hat{\pi}_t}_{\text{smoothing}} + \underbrace{\rho^k\frac{1-\beta}{\kappa\psi}\frac{\bar{\tau}}{s_d}(L_t^b - L_{t-1}^b)}_{\text{absorbing shock}},\ k \geq 1 \tag{72}$$

$$E_t\hat{x}_{t+k} = \underbrace{\rho^k\hat{x}_t}_{\text{smoothing}} + \underbrace{(1-\rho^k)\frac{1-\beta}{\lambda}(\frac{b_\tau}{\psi} - b_x)L_t^b}_{\text{absorbing shock}},\ k \geq 1 \tag{73}$$

Equations (72) and (73) are important to understand the effects of maturity structure on the formation of expectations. Two opposite intentions – smoothing to increase welfare and fluctuating to absorb shocks – both appear in determination of dynamics. The resulting evolution of inflation and output gap depends on how much weight being put on each intention. The weight is generally time-varying and determined by ρ^k (only $\rho = 0$ or 1 makes ρ^k not time-varying). For inflation, both weights on smoothing and shock absorbing are given by ρ^k, which is increasing in ρ and decreasing in k. Therefore, as the average maturity of debt rises, inflation is expected to be more smoothed and more responsive to exogenous shocks. However, this effect decays as time grows. In the time limit, expectation of future inflation will converge to zero, $\lim_{k\to\infty} E_t\pi_{t+k} = 0$. For output, the weight on smoothing is given by ρ^k, and the weight on shock absorbing is given by $1-\rho^k$. Therefore, as the average maturity of debt rises, output gap is expected to be more smoothed and less responsive to exogenous shocks. The time also matters. Given the average maturity, the further the period (the bigger the k), the less smoothing and more responsive to the exogenous shocks. In the limit, expectation of output gap will converge to a constant level, $\lim_{k\to\infty} E_t x_{t+k} = \frac{1-\beta}{\lambda}\frac{\sigma}{s_c}L_t^b$, a permanent deviation.

6 Calibration

We turn to numerical results from the model calibrated to U.S. data in order to focus on a set of implications that may apply to an actual economy.

Table 1 reports a calibration to U.S. time series. We take the model's frequency to be quarterly and adopt some parameter values from Benigno and Woodford (2004), including $\beta = 0.99$, $\theta = 0.66$ and $\epsilon = 10$; we set $\varphi = \sigma = 0.5$, implying a Frisch elasticity and an intertemporal elasticity of substitution of 2.0, both reasonable empirical values. Quarterly U.S. data from 1948Q1 to 2013Q1 underlie the values of s_b, s_g, s_z and are used to estimate autoregressive processes for A_t, G_t, τ_t, Z_t shocks.[17] Following Gali et al. (2007), the wage markup shock is calibrated as an AR(1) process with persistence of 0.95 and standard deviation of 0.054. Table 1's calibration makes the relative weight on output-gap stabilization equal to $\lambda = 0.0033$, slightly higher than the value used in Benigno and Woodford (2007) ($\lambda = 0.0024$).[18]

[17] Appendix H provides details.

[18] Benigno and Woodford's calibration of $\sigma = 0.16$ largely explains the difference in the values of λ.

parameter	definition	value
β	discount rate	0.99
σ	the inverse of intertemporal elasticity of substitution	0.50
φ	the inverse of Frisch elasticity of labor supply	0.50
θ	the fraction of firms cannot adjust their prices	0.66
ϵ	intratemporal elasticity of substitution across consumption goods	10
s_c	steady state consumption to gdp ratio	0.87
s_z	steady state government transfer payment to gdp ratio	0.09
s_g	steady state government spending-gdp ratio	0.13
s_b	steady state debt-gdp ratio	0.49×4
$\bar{\tau}$	steady state tax rate	0.24
ρ_a	autoregressive coefficient of tech shock	0.786
ρ_g	autoregressive coefficient of government spending shock	0.886
ρ_τ	autoregressive coefficient of tax rate shock	0.782
ρ_z	autoregressive coefficient of transfer payment shock	0.56
ρ_w	autoregressive coefficient of wage markup shock	0.95
σ_e^a	standard deviation of innovation to tech shock	0.008
σ_e^g	standard deviation of innovation to government spending shock	0.027
σ_e^τ	standard deviation of innovation to tax rate shock	0.029
σ_e^z	standard deviation of innovation to transfer payment shock	0.047
σ_e^w	standard deviation of innovation to wage markup shock	0.054

Table 1: Calibration to U.S. Data

7 Separating the Impacts of Maturity

To fully understand the impacts of debt maturity on the tradeoffs between inflation and output stabilization, we rewrite the government intertemporal equilibrium condition (30) in terms of only inflation and the output gap

$$\hat{b}_{t-1}^M + F_t = \underbrace{\hat{\pi}_t + (1-\beta)\frac{b_\tau}{\kappa\psi}\hat{\pi}_t}_{\text{surprise inflation}} + \underbrace{E_t\sum_{k=1}^{\infty}(\beta\rho)^k\hat{\pi}_{t+k}}_{\text{future inflation}}$$
$$- \underbrace{(1-\beta)b_\tau(\frac{1}{\psi}-1)E_t\sum_{k=0}^{\infty}\beta^k\hat{x}_{t+k}}_{\text{future output}} - \underbrace{E_t\sum_{k=0}^{\infty}[(1-\beta)\beta^k - (1-\beta\rho)(\beta\rho)^k]\frac{\sigma}{s_c}(\hat{x}_{t+k}-\hat{x}_t)}_{\text{real interest rate}} \tag{74}$$

Given the fiscal stress, $\hat{b}_{t-1}^M + F_t$, (74) completely summarizes feasible paths of current and expected inflation and output gaps. To absorb exogenous disturbances to F_t, some combination of paths of inflation and output must adjust. This equation underscores the inherent symmetry between monetary and fiscal policy: interactions between the two policies determine the reliance on variations in output gaps versus inflation rates.

Average duration of government bonds affects the optimal equilibrium through three channels. First, it affects overall fiscal stress, F_t, given the processes of exogenous disturbances. Second, it affects the allocation of inflation rates over time. Third, it affects real allocations through changing real discount rates.

7.1 ρ's Impact on F_t F_t, a composite measure of fiscal stress, summarizes the factors in our model that prevent complete stabilization of inflation and the welfare-relevant output gap. F_t may be expressed in terms of the four fundamental shocks

$$F_t = F_A\hat{A}_t + F_G\hat{G}_t + F_Z\hat{Z}_t + F_W\hat{\mu}_t^W \tag{75}$$

where

$$F_x = (1-\beta\rho_x)^{-1}[f_x + (1-\beta)\frac{\bar{\tau}}{s_d}\frac{1}{\kappa\psi}u_x + \beta\frac{\sigma}{s_c}v_x] - \beta\rho(1-\beta\rho\rho_x)^{-1}\frac{\sigma}{s_c}v_x, \quad x = A, G, Z, W \tag{76}$$

The average maturity of bonds affects the amount of fiscal stress imposed on equilibrium through the weights attached to each fundamental shock. Figure 1 plots the feedback coefficients for each of the fundamental shocks, as defined in (75). F_A is negative for all average bond durations, while F_G, F_Z, F_W are positive. A positive innovation to technology helps relieve fiscal stress by raising tax revenues, improving the tension between inflation and the output gap. Positive innovations to wage markups, government spending or transfer payments, in contrast, aggravate fiscal stress and make it more difficult to stabilize inflation and the output gap contemporaneously. The impacts of technology, wage markups and government spending shocks abate as the average maturity of bonds grows longer, while the impact of government transfers becomes stronger with longer-term bonds. In all case, though, the effects of the average maturity of bonds arise mainly when maturity extends from short-term (1 quarter) to medium-term (5 years); extending beyond 5 years does little. Finally, the impact of each fundamental shock on fiscal stress is ranked $|F_W| > |F_A| > |F_G| > |F_Z|$. Wage markups and technology shocks affect F_t more strongly than do government spending and transfers shocks. The complicated heterogeneity among different shocks in affecting the overall fiscal stress makes some of our results sensitive to the calibration of the four fundamental shocks.

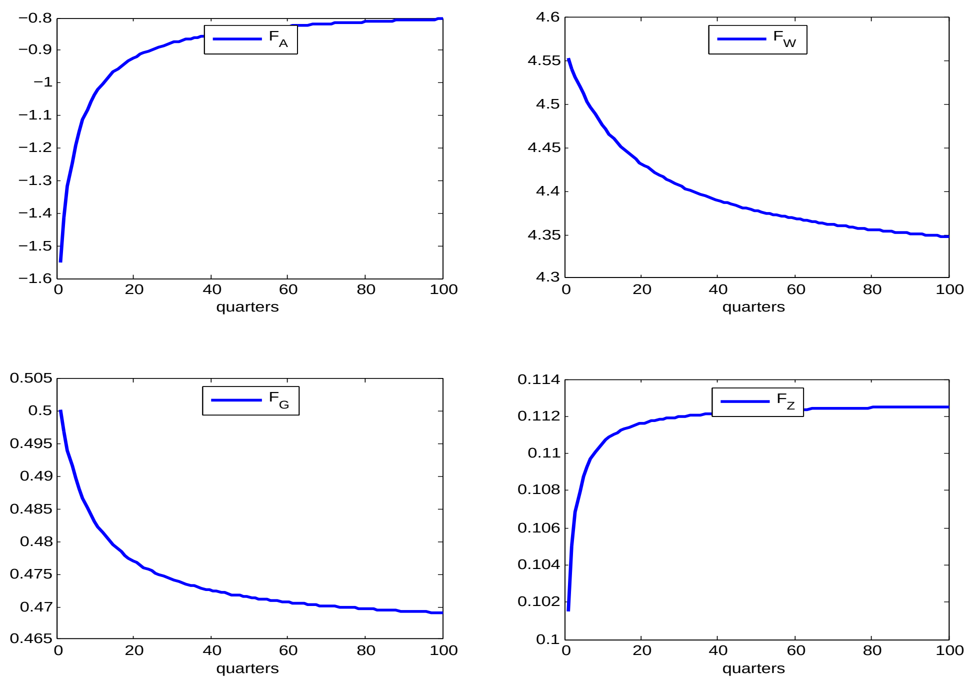

Figure 1: The mapping from fundamental shocks $\hat{A}_t$ (technology), $\hat{\mu}_t^W$ (wage markup), $\hat{G}_t$ (government purchases), $\hat{Z}_t$ (government transfers) to fiscal stress, F_t, in (75).

We turn now to study how the changes in average maturity of government bonds affects the tradeoff between stabilizing inflation and output gap. To distinguish the average maturity structure's impact on inflation and output gap, we consider two polar sub-optimal cases: i) complete stabilization of the output gap, depending only on inflation as a shock absorber and ii) complete stabilization of inflation, using only the output gap as a shock absorber.

7.2 Inflation Smoothing Complete output stabilization sets $\hat{x}_t \equiv 0$ for all $t \geq 0$ and uses current and future inflation to fully absorb innovations to $\hat{b}^M_{t-1} + F_t$. This polar case eliminates the effect of maturity on tax smoothing and to focus on how alternative maturity structures dynamically allocate inflation. Constraint (74) becomes

$$\hat{b}^M_{t-1} + F_t = \underbrace{\hat{\pi}_t + (1-\beta)\frac{b_\tau}{\kappa\psi}\hat{\pi}_t}_{\text{surprise inflation}} + \underbrace{E_t\sum_{k=1}^{\infty}(\beta\rho)^k\hat{\pi}_{t+k}}_{\text{bond prices}} \tag{77}$$

The term $(1-\beta)\frac{b_\tau}{\kappa\psi}\hat{\pi}_t$ stems from the effects of distorting taxes on inflation at time t. Recall that the Phillips curve

$$\hat{\pi}_t = E_t\sum_{k=0}^{\infty}\beta^k[\kappa\hat{x}_{t+k} + \kappa\psi(\hat{\tau}_{t+k} - \hat{\tau}^*_{t+k})]$$

implies that increasing future tax rates decreases the path of output or increases current inflation. When output is fully stabilized, inflation must rise. This inflationary effect of fiscal policy is absent from Cochrane (2001) and Sims (2013) and arises here from distortionary taxes.

Consider the following optimal inflation-smoothing problem

$$\min \ \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t q_\pi \hat{\pi}_t^2$$

$$s.t. \ \ \hat{b}^M_{t-1} + F_t = b_\pi\hat{\pi}_t + E_t\sum_{k=1}^{\infty}(\beta\rho)^k\hat{\pi}_{t+k} \tag{78}$$

where $b_\pi = 1 + (1-\beta)\frac{b_\tau}{\kappa\psi}$. First order condition yields

$$\hat{\pi}_t = b_\pi\lambda_t + (1-b_\pi)\rho\lambda_{t-1} \tag{79}$$

$$E_t\lambda_{t+1} = \rho\lambda_t \tag{80}$$

where λ_t is the multiplier associated with the intertemporal solvency condition, and it is known as the shadow price which measures the marginal change in the objective function arising from an infinitesimal change in the constraint. Condition (80) implies that the shadow price associated with fiscal stress is expected to decay at the rate of ρ.

In our model, if we set $b_\tau = 0$, eliminating the effects of distorting taxes in Phillips curve, then $b_\pi = 1$, condition (78), (79) and (80) imply

$$\hat{\pi}_t = (1 - \beta\rho^2)(\hat{b}^M_{t-1} + F_t)$$
$$E_t\hat{\pi}_{t+k} = \rho^k \hat{\pi}_t, \quad k \geq 1$$

Thus unexpected changes in the fiscal stress term must be accommodated entirely by surprise variations in current and future inflation. The immediate response of inflation at time t is increasing in the fiscal stress F_t and decreasing in the average maturity ρ. The longer the average maturity of government bonds, the smaller the immediate response of inflation. Future inflations are expected to decay at the rate of ρ. When $\rho = 1$, perfect smoothing in inflation $E_t\hat{\pi}_{t+k} = \hat{\pi}_t$, as in Sims (2013).

In general, taking expectation of (79) and applying (80), we obtain

$$E_t\hat{\pi}_{t+1} = \frac{\rho}{b_\pi}\hat{\pi}_t + \frac{b_\pi - 1}{b_\pi}\rho^2\lambda_{t-1} \quad \text{and} \quad E_t\hat{\pi}_{t+k+1} = \rho E_t\hat{\pi}_{t+k}, \quad k \geq 1$$

If $\rho = 0$, we have $E_t\hat{\pi}_{t+k} = 0$ $(k \geq 1)$, inflation is expected to remain at zero. If $\rho = 1$, we have $E_t\hat{\pi}_{t+k} = \frac{\hat{\pi}_t}{b_\pi} + \frac{b_\pi - 1}{b_\pi}\lambda_{t-1}$ $(k \geq 1)$, inflation is expected to deviate permanently by a constant amount. If $0 < \rho < 1$, inflation is expected to deviate by $\frac{\rho}{b_\pi}\hat{\pi}_t + \frac{b_\pi - 1}{b_\pi}\rho^2\lambda_{t-1}$ at period t+1 and then decay at rate of ρ afterwards. The immediate response of inflation at period t is given by

$$\hat{\pi}_t = \frac{\hat{b}^M_{t-1} + F_t - \frac{\beta\rho^3}{1-\beta\rho^2}\frac{b_\pi - 1}{b_\pi}\lambda_{t-1}}{b_\pi + \frac{\beta\rho^2}{1-\beta\rho^2}b_\pi^{-1}} \tag{81}$$

Figure 2 plots responses of inflation to a unit innovation in fiscal stress, F_t. With only one-period bonds, inflation jumps immediately and then returns to zero, since intertemporal smoothing of inflation is unavailable. With 5-year bonds, inflation reacts less aggressively in the first period, and then gradually goes back to zero. The presence of long-term bonds allows the government to tradeoff inflation today for inflation in the future. Finally, with consols, the immediate response of inflation is the smallest, with future inflation permanently, but only slightly, higher.[19]

[19] This result differs slightly from Sims (2013), who finds complete smoothing of inflation with only consols. Differences stem from Sims's use of lump-sum taxes, which do not have the direct inflationary effects that distortionary taxes produce.

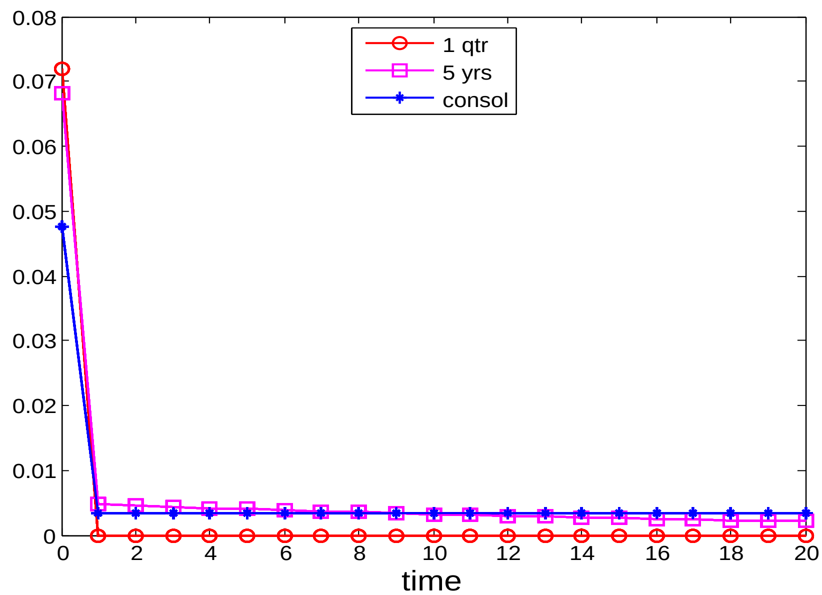

Figure 2: Inflation responses to a one unit innovation in fiscal stress

7.3 Output Smoothing Suppose now that $\hat{\pi}_t \equiv 0$ so that adjustments in current and future output gaps must absorb innovations to $\hat{b}^M_{t-1} + F_t$. Then (74) becomes

$$\hat{b}^M_{t-1} + F_t = \underbrace{-(1-\beta)\left(\frac{b_\tau}{\psi} - b_x\right) E_t \sum_{k=0}^{\infty} \beta^k \hat{x}_{t+k}}_{\text{tax revenues}} + \underbrace{\frac{\sigma}{s_c}(1-\beta\rho) E_t \sum_{k=0}^{\infty} (\beta\rho)^k \hat{x}_{t+k}}_{\text{bond price}} \qquad (82)$$

Changes in output help to satisfy the solvency condition by directly affecting tax revenues and by the impacts of output on real returns to the bond portfolio. These two parts produce opposite effects. Consider an exogenous increase in fiscal stress. Relying on tax revenues to fully offset this disturbance requires increasing the tax rate. But by the Phillips curve

$$\hat{\pi}_t = E_t \sum_{k=0}^{\infty} \beta^k [\kappa \hat{x}_{t+k} + \kappa\psi(\hat{\tau}_{t+k} - \hat{\tau}^*_{t+k})]$$

when $\pi_t \equiv 0$, a higher tax rate must reduce output. This is the negative effect in the first term. Lower output generates a second effect by increasing real returns on long-term bonds. This positive effect is captured by the second term. The second effect is usually omitted by assuming a constant or exogenous real discount rate as in Cochrane (2001) and Sims (2013). In our model, if we set $\sigma = 0$, then utility is linear and the real rate is constant. This mutes the second channel and we obtain a smoothing of the output gap $E_t \hat{x}_{t+1} = \hat{x}_t$ as in Sims (2013).

Consider the following optimal output-gap smoothing problem:

$$\min \ \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t q_x \hat{x}_t^2$$
$$s.t.\ \hat{b}_{t-1}^M + F_t = m_x E_t\sum_{k=0}^{\infty}(\beta\rho)^k\hat{x}_{t+k} - n_x E_t\sum_{k=0}^{\infty}\beta^k\hat{x}_{t+k} \tag{83}$$

where $m_x = (1-\beta\rho)\frac{\sigma}{s_c}$ and $n_x = (1-\beta)(\frac{b_\tau}{\psi} - b_x)$. The first-order condition yields

$$\hat{x}_t = m_x(\mu_t - \mu_{t-1}) - n_x(\mu_t - \rho\mu_{t-1}) \tag{84}$$
$$E_t(\mu_{t+1} - \mu_t) = \rho(\mu_t - \mu_{t-1}) \tag{85}$$
$$E_t(\mu_{t+1} - \rho\mu_t) = \mu_t - \rho\mu_{t-1} \tag{86}$$

where μ_t is the multiplier associated with the intertemporal solvency condition (83). Condition (84) implies that the output gap can be expressed by two terms, the first term $(\mu_t - \mu_{t-1})$ is expected to decay at rate of ρ according to (85) ; the second term $(\mu_t - \rho\mu_{t-1})$ is expected to be perfectly smoothed according to (86). The dynamics of output gap depends on the relative weight on each term – m_x and n_x. Note that m_x is decreasing in ρ and n_x is constant, therefore, the longer the average maturity, the smaller weight on the decaying term, the more smoothing in output gap. When $\sigma = 0$, $m_x = 0$, therefore output gap is perfectly smoothed.

Furthermore, we can express the expectation of output gap as

$$E_t\hat{x}_{t+k} = \hat{x}_t + m_x(\rho^k - 1)(\mu_t - \mu_{t-1}), \quad k \geq 1$$

Therefore, the difference of output gaps between two periods $\hat{x}_{t+k} - \hat{x}_t$ (for any $k \geq 1$) is expected to decay at rate of ρ. If $\rho = 0$, we have $E_t\hat{x}_{t+k} = \hat{x}_t - \frac{\sigma}{s_c}(\mu_t - \mu_{t-1})$ ($k \geq 1$), output gap is expected to deviate permanently by a constant amount. If $\rho = 1$, we have $E_t\hat{x}_{t+k} = x_t$ ($k \geq 1$), output gap is expected to be perfectly smoothed out. The immediate response of output gap at period t is given by

$$\hat{x}_t = \frac{\hat{b}_{t-1}^M + F_t - c_x(\mu_t - \mu_{t-1})}{(1-\psi^{-1})b_\tau} \tag{87}$$

where $c_x = \beta[\frac{(1-\rho)^2(\frac{\sigma}{s_c})^2}{1-\beta\rho^2} + (1-\rho)\frac{\sigma}{s_c}(\psi^{-1}-1)b_\tau]$.

Figure 3 plots responses of output-gap to a unit innovation in fiscal stress, F_t. With only one-period bonds, the output gap first jumps by a large amount at time 0 and then drop

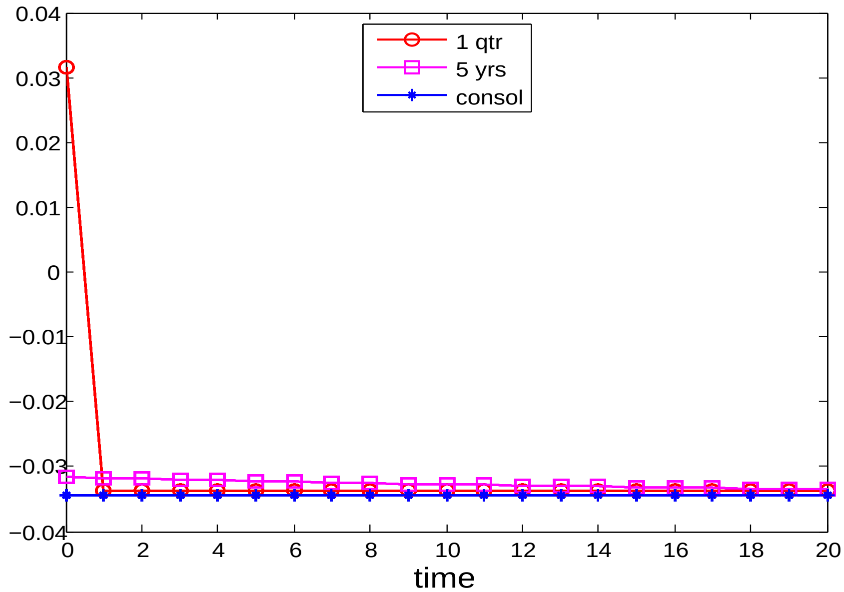

Figure 3: Output responses to a one unit innovation in fiscal stress

to a constant level. With 5-year bonds, output drops in the first period and then gradually converges to the same level as with one-period bonds. With consols, the output gap is perfectly smoothed. Long-term bonds allow the government to smooth the output gap.

8 Optimal Choice Between Inflation and Output Stabilization

This section brings together inflation smoothing with output gap smoothing to examine the joint determination of output and inflation in the presence of distortionary taxes and sticky prices. With distortionary taxes, variations in tax rates lead to distortions in real allocations and generates welfare loss; with sticky prices, unexpected variations in inflation create distortions in the allocation of resources and reduces welfare. If there is a disturbance to the government budget, monetary and fiscal policy face a tradeoff between stabilizing inflation and stabilizing the output gap. Panel III in figure **??** plots the impulse responses under the fully optimal solution to a unit increase in fiscal stress. Responses are much like a combination of responses in panels I and II. Both inflation and output adjust to absorb disturbances to F_t.

The optimum problem minimizes the loss function

$$L = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (q_\pi \hat{\pi}_t^2 + q_x \hat{x}_t^2) \tag{88}$$

subject to the constraint given by (74)

$$\hat{b}^M_{t-1} + F_t = \hat{\pi}_t + (1-\beta)\frac{b_\tau}{\kappa\psi}\hat{\pi}_t + E_t\sum_{k=1}^{\infty}(\beta\rho)^k\hat{\pi}_{t+k}$$
$$- (1-\beta)b_\tau(\frac{1}{\psi}-1)E_t\sum_{k=0}^{\infty}\beta^k\hat{x}_{t+k} - E_t\sum_{k=0}^{\infty}[(1-\beta)\beta^k - (1-\beta\rho)(\beta\rho)^k]\frac{\sigma}{s_c}(\hat{x}_{t+k}-\hat{x}_t)$$

Solving this problem yields the optimal equilibrium paths of inflation and the output gap. Defining $L_\pi = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t q_\pi \pi_t^2$ and $L_x = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t q_x x_t^2$, we can decompose the loss function into two parts

$$L = L_\pi + L_x \tag{89}$$

where L_π measures the welfare loss from fluctuations in inflation (expected present value of welfare losses associated with inflation variability) and L_x measures the welfare loss from fluctuations in output (expected present value of welfare losses associated with output-gap variation).[20] For each value of average maturity ρ, we compute the value of loss function L and the optimal mix of L_π and L_x, which we plot in the upper panel of figure 4.

The left panel of figure 4 plots the value of objective loss L as a function of ρ. The loss function is hump-shaped in ρ: when the average maturity increases from 1 to 2 quarters, the objective loss L, expressed in terms of the equivalent permanent consumption decline, increases by 1 percentage point; after 2 quarters, the loss L decreases monotonically in ρ. To see why the loss function is hump-shaped, return to figure **??**. When the average maturity is relatively short, the output gap tends to be volatile, increasing welfare losses. Our conclusion is consistent with Eusepi and Preston (2012), who find that medium average maturity is most harmful for stability.

In the right panel of figure 4, the present value of welfare losses for inflation variation, L_π, is plotted along the horizontal axis and the losses from output fluctuations, L_x, is plotted along the vertical axis. $L = L_\pi + L_x$ represents negatively sloped isoloss lines. Isoloss lines closer to the origin correspond to lower loss. The optimal mix of L_π and L_x is plotted as shaded circles as ρ varies. For all maturities, L_x is almost 10 times larger than L_π, implying that inflation is better stabilized than is the output gap. Other dynamic patterns emerge. As the average maturity of bonds moves from 1 to 2 quarters, both L_π and L_x increase, consistent with the increase in the overall loss function. As average maturity moves from 2

[20] We define L_x and L_π using second moment of $\hat{\pi}_t$ and $\hat{x}_t$ intead of computing uncontional variance of $\hat{\pi}_t$ and $\hat{x}_t$, because with $\rho > 0$, $\hat{x}_t$ and $\hat{\pi}_t$ might have unit root, and the variances are time-varying. This is analogous to Taylor's (1979) policy tradeoff curves.

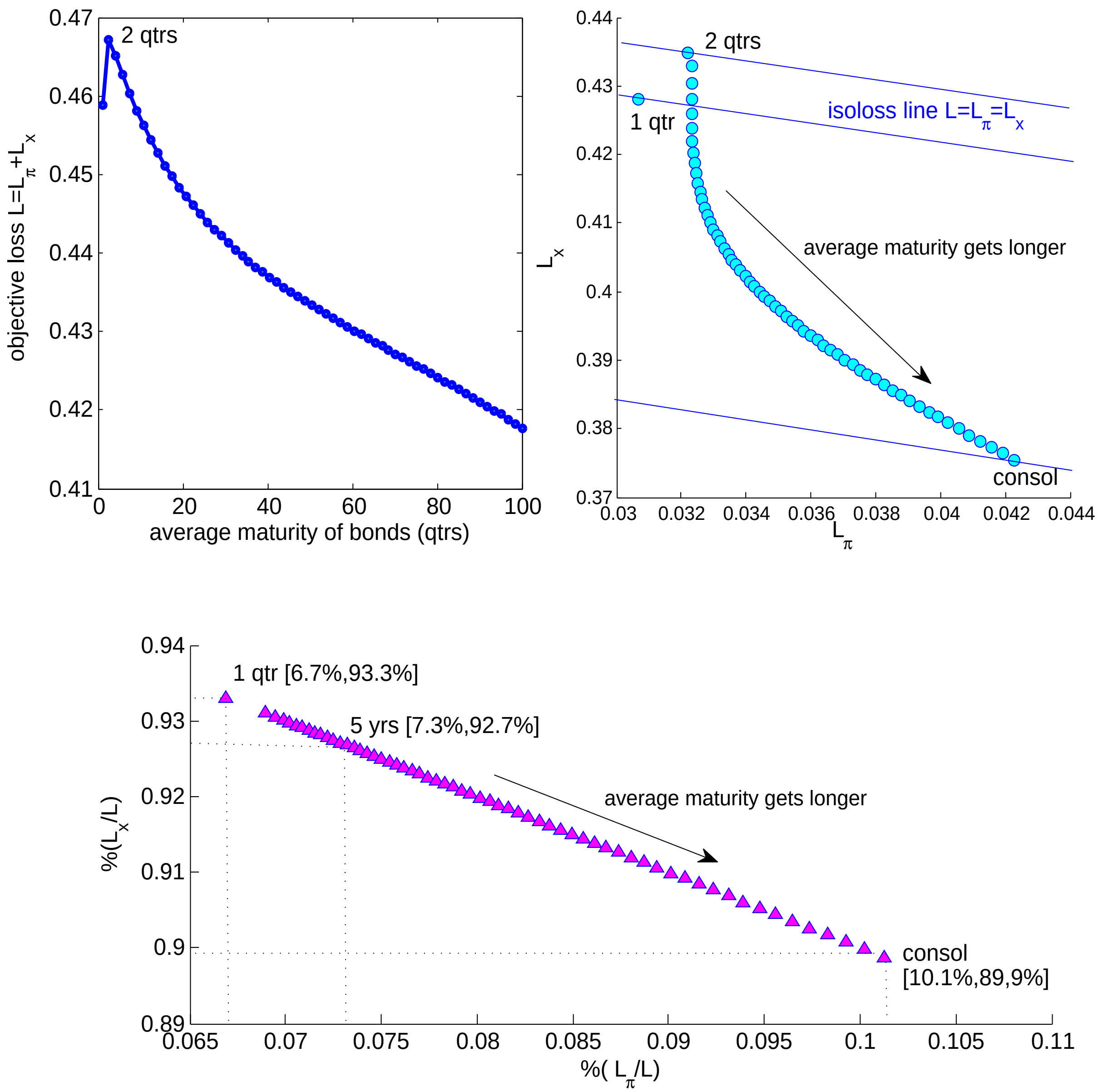

Figure 4: Welfare losses—total and due to inflation and output variability—as a function of average duration of government debt, expressed as percentages of steady state consumption

quarters toward consols, L_x decreases and L_π generally increases: stabilization of the output gap becomes more desirable from a welfare perspective as the average maturity of debt extends.

To shed light on the relative importance of inflation versus output variation as sources of welfare losses, the lower panel of figure 4 plots the ratios $\frac{L_\pi}{L}$ and $\frac{L_x}{L}$ for different average maturities. When the average maturity is 1 quarter, L_π accounts for 6.7% of overall loss while L_x accounts for 93.3% of overall loss. As the average maturity gets longer, L_π accounts for a larger fraction of the welfare losses. In the case of consols, L_π accounts for 10% of overall loss while L_x accounts for 90%. There is a strictly increasing trend in depending on fluctuations in inflation to hedge against exogenous shocks when average maturity of government debt extends.

9 Fiscal Financing

Sims (2013) emphasizes the role of surprise inflation as a "fiscal cushion" that can reduce the reliance on distorting sources of revenues. One way to quantify the fiscal cushion is to use the government's solvency condition to account for the sources of fiscal financing—including current and future inflation—following an innovation in the present value of fiscal stress F_t. The government solvency condition may be written as

$$\hat{b}^M_{t-1} + F_t = \underbrace{\hat{\pi}_t + (1-\beta)\frac{b_\tau}{\kappa\psi}\hat{\pi}_t}_{\text{surprise inflation}} + \underbrace{E_t\sum_{k=1}^{\infty}(\beta\rho)^k\hat{\pi}_{t+k}}_{\text{future inflation}} \\ \underbrace{- (1-\beta)b_\tau(\psi^{-1}-1)E_t\sum_{k=0}^{\infty}\beta^k\hat{x}_{t+k}}_{\text{future output}} \underbrace{- E_t\sum_{k=0}^{\infty}[(1-\beta)\beta^k-(1-\beta\rho)(\beta\rho)^k]\frac{\sigma}{s_c}(\hat{x}_{t+k}-\hat{x}_t)}_{\text{real interest rate}} \tag{90}$$

Fiscal financing underscores the inherent symmetry between monetary and fiscal policy: interactions between the two policies determine the reliance on tax revenues, which decreases output, versus current and future inflation.

Figure 5 plots the financing decomposition for one unit increase of fundamental shock as a function of the average duration of government bonds for the calibration to U.S. data in table 1. The pattern of this decomposition is robust in that it does not depend on the nature of shocks. In figure 5 the vast majority of financing comes from a decrease in output. As the average duration increases, $\rho \to 1$, the importance of real interest rate adjustments dissipates. In the new Keynesian model, real interest rates transmit immediately into movements in the output gap, so at short durations, distortions in output are relatively big. As duration rises, it is optimal to smooth output more, so real interest rate movements diminish. In the limit, when $\rho = 1$, the present value of real interest rates is zero and it is optimal to make $E_t\hat{x}_{t+1} = \hat{x}_t$ and rely instead on inflation as a fiscal cushion.

Sources of fiscal financing are particularly sensitive to the level of debt in the economy. Figure 6 reports fiscal financing decompositions under three steady state debt-GDP levels:

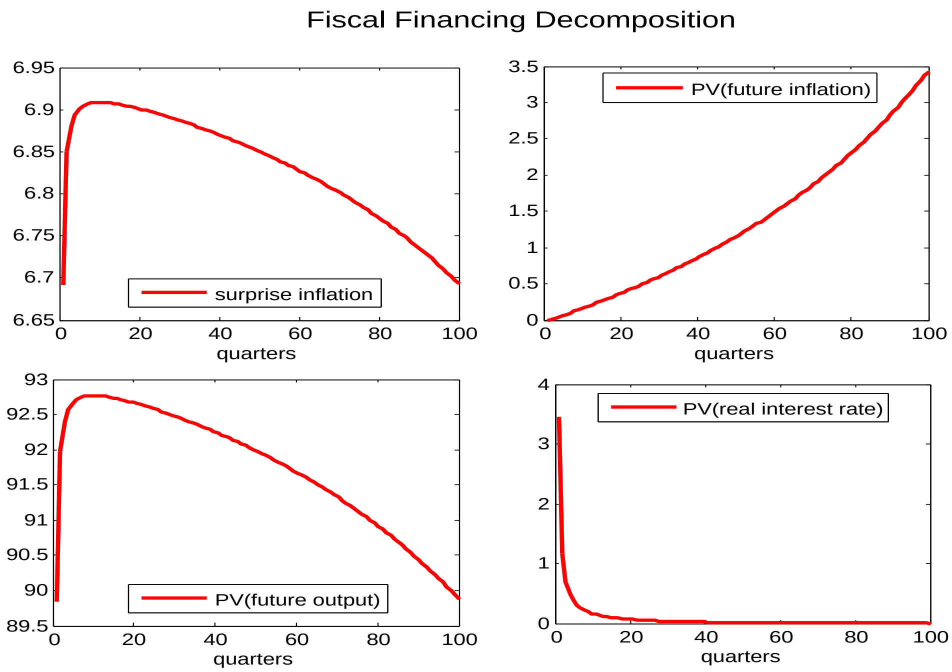

Figure 5: Fraction of fiscal stress innovation financed by each of the four components (90), as a function of average duration of government debt

the calibration to U.S. data (49 percent), "low debt" (20 percent), "high debt" (100 percent). As the level of debt rises, the reliance on tax financing declines. With very short debt duration, changes in real interest rates account for a substantial fraction of financing in high-debt economies. Reliance on real rates declines rapidly as duration rises, with future inflation becoming increasingly important. With long-duration debt, high-debt economies would finance over 20 percent of a fiscal stress innovation with current and future inflation.

10 Contrast to Conventional Optimal Monetary Policy

The conventional optimal monetary policy problem, as Woodford (2011) describes, typically assumes a nondistorting source of revenue exists, so that stabilization policy abstracts from fiscal policy distortions.[21] To place the conventional optimal problem on an equal footing with the fully optimal problem, we have the government optimally choose the interest rate function, taking as given exogenous processes for technology, government spending, the distorting tax rate and wage markup; lump-sum transfers (or taxes) adjust passively to ensure the government's solvency condition never binds. In the conventional problem, the maturity structure of debt is irrelevant. In this section we contrast fully optimal policy to the con-

[21] Key earlier expositions of the conventional optimal monetary policy problem include Clarida et al. (1999b) and Woodford (2003).

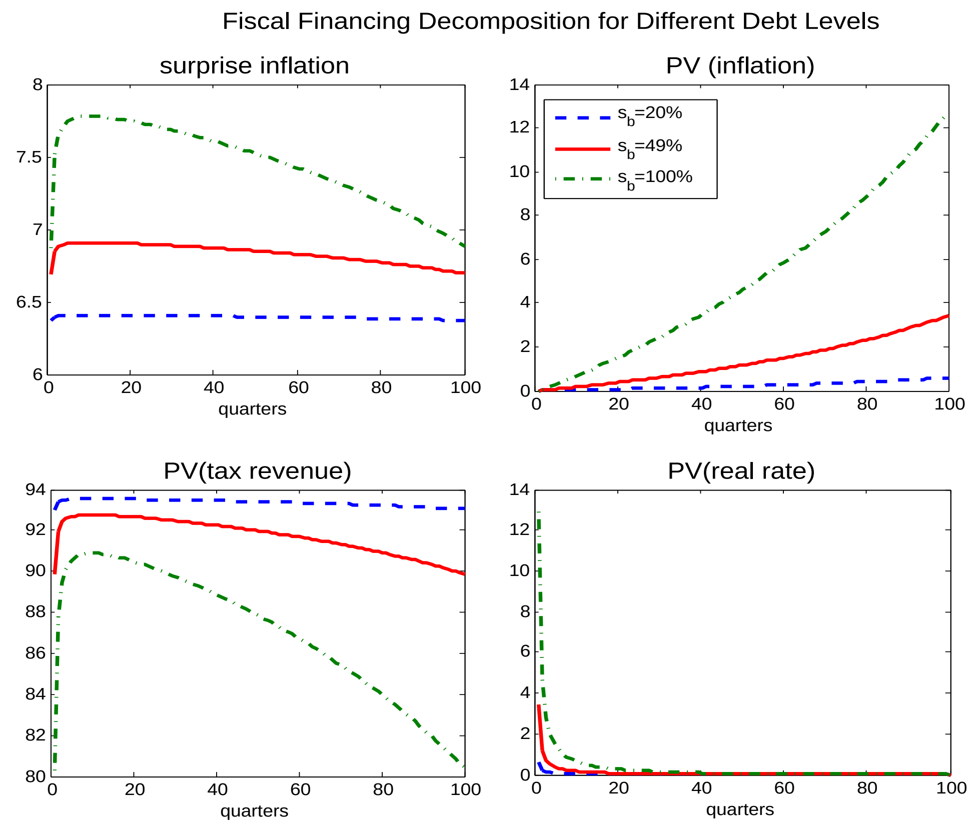

Figure 6: Fraction of fiscal stress innovation financed by each of the four components (90), as a function of average duration of government debt, for three steady state debt-output ratios

ventional optimal monetary policy. In both cases, we examine the case of a distorted steady state.

Figure 7 plots the value of the loss function under the conventional optimal monetary policy—straight yellow line—and under the fully optimal monetary and fiscal policies—dotted red line—as a function of the average duration of government debt. These calculations employ the calibration in table 1 for U.S. data. The value of loss objective function under fully optimal polices is hump-shaped in ρ, and even though fully optimal policies do not use lump-sum taxes to make the government solvency condition non-binding, welfare is higher under fully optimal policies if the average maturity is long enough (longer than 21 years). The figure shows that welfare under the two optimal policy regimes can be made equivalent by extending the average maturity of bond. However, this welfare-equivalence result and the level of average maturity that achieves the the welfare-equivalence might be sensitive to our calibration.

Figure 8 reports the implications of average debt levels for the threshold value of average maturity that achieves equivalent welfares between fully optimal policies and conventional optimal monetary policy. We consider the range of [20%, 100%], which covers most countries' debt-to-GDP ratios in the world. We see that when the debt-to-GDP ratio is at low level (below 40%), no average maturity exists that achieves equivalent welfares: welfare under fully optimal polices is always inferior to the conventional optimal monetary policy with lump-sum taxes, no matter how long the average maturity. It is possible to achieve welfare equivalence only in medium- to high- debt economies, that is, only when debt-to-GDP ratio is higher than 40%. Moreover, the higher the debt levels, the shorter the threshold value of average maturity to achieve welfare equivalence.

Figure 9 reports the implications of standard deviations of wage markup shock for the threshold value of average maturity that achieves equivalent welfares between fully optimal policies and conventional optimal monetary policy. We consider the range of [0.02, 0.06], which covers most calibrations of standard deviation of wage markup in the literatures.

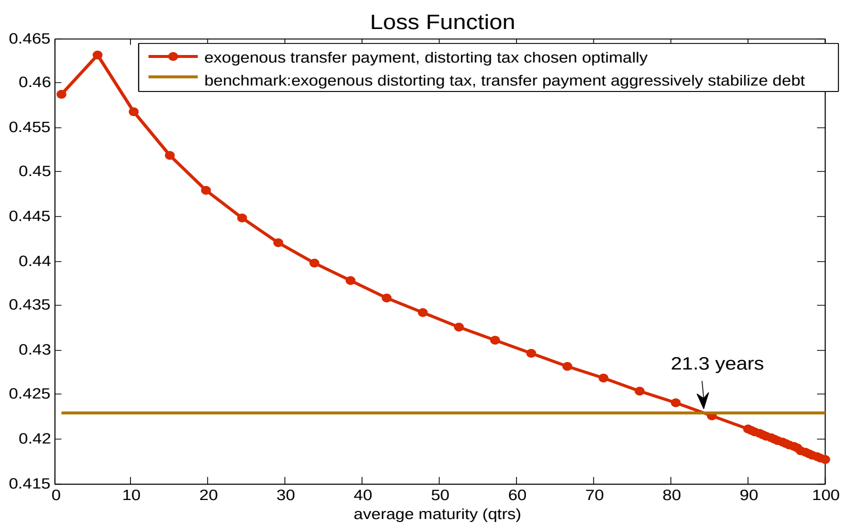

Figure 7: Value of loss function as a percentage of steady state consumption

11 Concluding Remarks

This paper examines the joint determination of optimal monetary and fiscal policy in the presence of distorting tax and sticky prices. We study how the presence of long-term bonds affects optimal allocations between inflation and output gap, and the consequent stabilization role for monetary and fiscal policy.

We identified three channels for long-term bonds to have effect on optimal allocations between inflation and output gap. First, long-term bonds affect the aggregate fiscal stress imposed on the intertemporal solvency condition that prevents complete stabilization of inflation and welfare-relevant output gap. Second, long-term bonds facilitate intertemporal

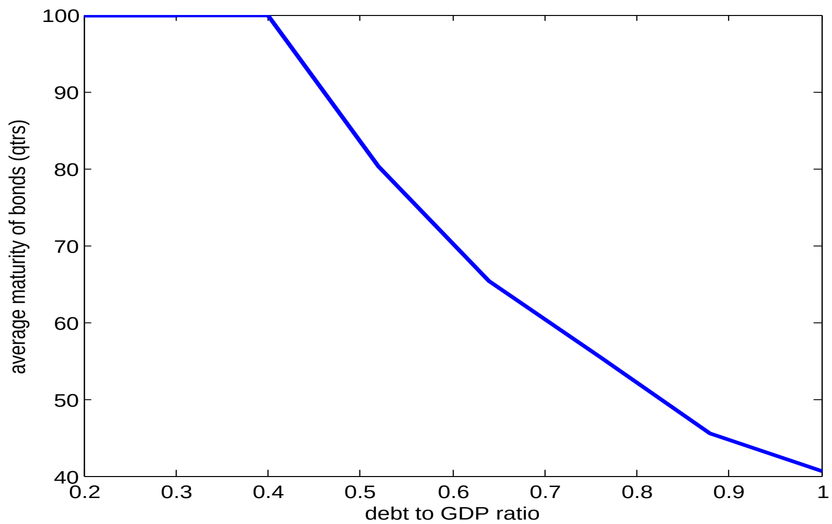

Figure 8: Threshold level of average maturity that equates welfare under fully optimal monetary and fiscal policies and welfare under conventional optimal monetary policy with passively adjusting lump-sum taxes, as a function of debt-output ratio

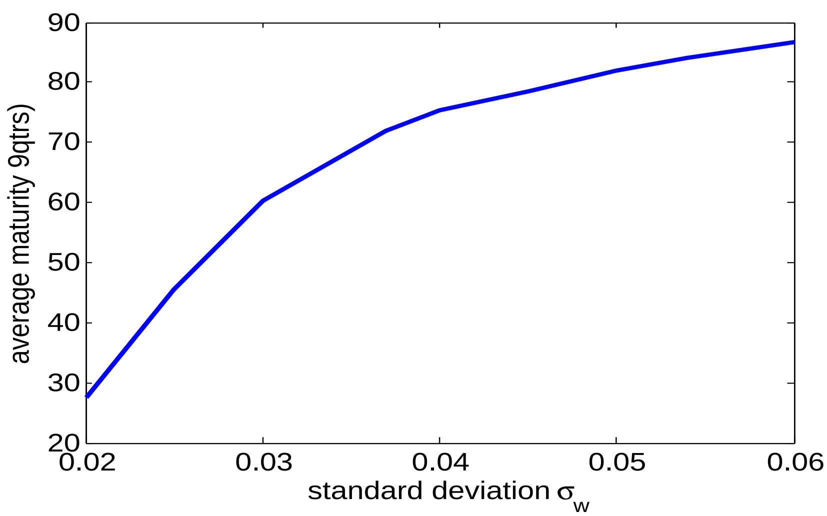

Figure 9: Threshold level of average maturity that equates welfare under fully optimal monetary and fiscal policies and welfare under conventional optimal monetary policy with passively adjusting lump-sum taxes, as a function of the standard deviation of wage markup shock

smoothing for inflation. Third, which is new in our paper, long-term bond also smoothes welfare-relevant output gap by smoothing real interest rates.

To study inflation's role as a "fiscal cushion," we use the government's solvency condition to account for the sources of fiscal financing. As the duration of government debt rises, it is optimal to smooth output more and to rely on current and future inflation innovations to revalue government bond. In the limit, with only consol debt, it is optimal to perfectly smooth real interest rates and rely instead on inflation as a fiscal cushion. Sources of fiscal financing are also sensitive to the level of debt in the economy. As the level of bonds rises, the reliance on tax financing declines. With long-duration bond, high-debt economies (100% debt to GDP ratio) would finance over 20 percent of a fiscal stress innovation with current and future inflation.

Finally, we contrast the welfare under fully optimal policy to the conventional optimal monetary policy case where lump-sum taxes are available to always guarantee government solvency. We show that welfare under the two optimal policy regimes can be made equivalent by extending the average maturity of bond.

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A Derivation of Long-term Bond Price and IEC

Define

$$Q_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} \tag{A.1}$$

as the stochastic discount factor for the price at t of one unit of composite consumption goods at $t+k$. Then (2) and (3) in the text can be written as

$$Q_t^S = E_t Q_{t,t+1} \tag{A.2}$$

$$Q_t^M = E_t Q_{t,t+1}(1 + \rho Q_{t+1}^M) \tag{A.3}$$

Iterating on (A.3) and imposing a terminal condition yields

$$\begin{aligned} Q_t^M =& E_t[Q_{t,t+1} + \rho Q_{t,t+1} Q_{t+1}^M] \\ =& E_t\{Q_{t,t+1} + \rho Q_{t,t+1} E_{t+1} Q_{t+1,t+2} + \rho^2 Q_{t,t+1} E_{t+1}[Q_{t+1,t+2} E_{t+2} Q_{t+2,t+3}] + ...\} \\ =& Q_t^S + \rho E_t[Q_{t,t+1} Q_{t+1}^S] + \rho^2 E_t[Q_{t,t+1} E_{t+1}(Q_{t+1,t+2} Q_{t+2}^S)] + ... \\ =& Q_t^S + \rho E_t[Q_{t,t+1} Q_{t+1}^S] + \rho^2 E_t[Q_{t,t+2} Q_{t+2}^S] + ... + \rho^k E_t[Q_{t,t+k} Q_{t+k}^S] + ... \\ =& Q_t^S + E_t \sum_{k=1}^{\infty} \rho^k E_t[Q_{t,t+k} Q_{t+k}^S] \end{aligned} \tag{A.4}$$

Equation (A.4) implies that the long-term bond's price is determined by weighted average of expectations of future short-term bond's prices.

Substitute (A.1) into (A.4)

$$\begin{aligned} Q_t^M &= E_t \; [\beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} + \rho \beta^2 \frac{U_{c,t+2}}{U_{c,t}} \frac{P_t}{P_{t+2}} + ... + \rho^{k-1} \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} + ...] \\ &= E_t \sum_{k=1}^{\infty} \rho^{k-1} \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}} \end{aligned} \tag{A.5}$$

Condition (A.5) implies that the long-term bond price is determined by the whole path of expected future price level, discounted by consumption growth rate. The long-term bond price is negatively correlated with expected future inflation rate and consumption growth rate.

Rewrite (A.3) as

$$\begin{aligned} Q_t^M =& E_t Q_{t,t+1}(1 + \rho Q_{t+1}^M) \\ =& E_t Q_{t,t+1} E_t(1 + \rho Q_{t+1}^M) + \rho \; cov(Q_{t,t+1}, Q_{t+1}^M) \\ =& E_t Q_t^S(1 + \rho Q_{t+1}^M) + \rho \; cov(Q_{t,t+1}, Q_{t+1}^M) \end{aligned} \tag{A.6}$$

Recall from (A.5) that

$$Q_{t+1}^{M} = E_t \sum_{k=1}^{\infty} \rho^{k-1} Q_{t+1,t+1+k}$$

Q_{t+1}^{M} is determined by weighted average of expected future discounted value of future stochastic discount factors. Therefore, without loss of generality, we assume $cov(Q_{t,t+1}, Q_{t+1}^{M}) = 0$, and (A.6) can be expressed as

$$Q_t^M = E_t Q_t^S (1 + \rho Q_{t+1}^M) \tag{A.7}$$

To derive intertemporal equilibrium condition, we iterate on government's period budget constraint, (9), and impose asset-pricing relations and the household's transversality condition:

$$\begin{aligned}(1+\rho Q_t^M)\frac{B_{t-1}^M}{P_t} &= Q_t^M \frac{B_t^M}{P_t} + S_t \\ &= E_t[\frac{Q_t^M Q_{t+1}^M \pi_{t+1}}{1+\rho Q_{t+1}^M}\frac{B_t^M}{P_t} + \frac{Q_t^M \pi_{t+1}}{1+\rho Q_{t+1}^M} S_{t+1} + S_t] \\ &= E_t[S_t + \frac{Q_t^M \pi_{t+1}}{1+\rho Q_{t+1}^M} S_{t+1} + \frac{Q_t^M Q_{t+1}^M \pi_{t+1}\pi_{t+2}}{(1+\rho Q_{t+1}^M)(1+\rho Q_{t+2}^M)} S_{t+2} + ...]\end{aligned} \tag{A.8}$$

Substituting (A.1) and (A.3) into (A.8) yields

$$(1+\rho Q_t^M)\frac{B_{t-1}^M}{P_t} = E_t \sum_{i=0}^{\infty} \beta^i \frac{U_{c,t+i}}{U_{c,t}} S_{t+i} \tag{A.9}$$

To derive (11) in the text, combine (A.5) and (A.9)

$$E_t \left(\sum_{k=0}^{\infty} \rho^k \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{1}{P_{t+k}} \right) B_{t-1}^M = E_t \sum_{i=0}^{\infty} \beta^i \frac{U_{c,t+i}}{U_{c,t}} S_{t+i} \tag{A.10}$$

B Derivation of Nonlinear First Order Conditions

The full nonlinear optimal policy problem maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, \xi_t)$$

subject to

$$[\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}]^{\frac{1+\epsilon\varphi}{1-\epsilon}} = \frac{\epsilon}{\epsilon-1}\frac{K_t}{J_t} \tag{B.1}$$

$$K_t = (\frac{Y_t}{A_t})^{\varphi+1} + \beta\theta E_t K_{t+1}\pi_{t+1}^{\epsilon(1+\varphi)} \tag{B.2}$$

$$J_t = (1-\tau_t)U_{c,t}Y_t + \beta\theta E_t J_{t+1}\pi_{t+1}^{\epsilon-1} \tag{B.3}$$

$$(1+\rho Q_t^M)\frac{b_{t-1}^M}{\pi_t} = Q_t^M b_t^M + \tau_t Y_t - Z_t - G_t \tag{B.4}$$

$$Q_t^M = \beta E_t \frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}(1+\rho Q_{t+1}^M) \tag{B.5}$$

$$\Delta_t = (1-\theta)[\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}]^{\frac{\epsilon(1+\varphi)}{\epsilon-1}} + \theta\pi_t^{\epsilon(1+\varphi)}\Delta_{t-1} \tag{B.6}$$

The first-order conditions are

$$\begin{aligned}
Y_t:\ & U_{Y,t} - \lambda_t^2(\varphi+1)A_t^{-(\varphi+1)}Y_t^{\varphi} - \lambda_t^3(1-\tau_t)(U_{c,t}+U_{cc,t}Y_t) - \lambda_t^4\tau_t \\
& -\beta\sigma\lambda_t^5\frac{1+\rho Q_{t+1}}{\pi_{t+1}}C_{t+1}^{-\sigma}C_t^{\sigma-1} + \sigma\lambda_{t-1}^5\frac{1+\rho Q_t}{\pi_t}C_{t-1}^{\sigma}C_t^{-\sigma-1} = 0 \\
\pi_t:\ & \lambda_t^1\frac{1+\epsilon\varphi}{1-\epsilon}p(\pi_t)^{\frac{1+\epsilon\varphi}{1-\epsilon}-1}p'(\pi_t) - \lambda_{t-1}^2\theta\epsilon(1+\varphi)K_t\pi_t^{\epsilon(1+\varphi)-1} - \lambda_{t-1}^3\theta(\epsilon-1)J_t\pi_t^{\epsilon-2} \\
& -\lambda_t^4(1+\rho Q_t^M)b_{t-1}\pi_t^{-2} + \lambda_{t-1}^5\frac{U_{c,t}}{U_{c,t-1}}(1+\rho Q_t^M)\pi_t^{-2} \\
& -\lambda_t^6[(1-\theta)\frac{\epsilon(1+\varphi)}{\epsilon-1}p(\pi_t)^{\frac{1+\epsilon\varphi}{\epsilon-1}}p'(\pi_t) + \theta\epsilon(1+\varphi)\Delta_{t-1}\pi_t^{\epsilon(1+\varphi)-1}] = 0 \\
\Delta_t:\ & U_{\Delta,t} + \lambda_t^6 - E_t\lambda_{t+1}^6\beta\theta\pi_{t+1}^{\epsilon(1+\varphi)} = 0 \\
K_t:\ & -\lambda_t^1\frac{\epsilon}{\epsilon-1}\frac{1}{J_t} + \lambda_t^2 - \lambda_{t-1}^2\theta\pi_t^{\epsilon(1+\varphi)} = 0 \\
J_t:\ & \lambda_t^1\frac{\epsilon}{\epsilon-1}\frac{K_t}{J_t^2} + \lambda_t^3 - \lambda_{t-1}^3\theta\pi_t^{\epsilon-1} = 0 \\
\tau_t:\ & \lambda_t^3 U_{c,t} - \lambda_t^4 = 0 \\
b_t:\ & \lambda_t^4 - E_t\lambda_{t+1}^4\frac{U_{c,t}}{U_{c,t+1}} = 0 \\
Q_t^M:\ & \lambda_t^4(\rho\frac{b_{t-1}}{\pi_t} - b_t) + \lambda_t^5 - \lambda_{t-1}^5\frac{\rho}{\pi_t}\frac{U_{c,t}}{U_{c,t-1}} = 0
\end{aligned}$$

If we redefine $\tilde{\lambda}_t^1 = \frac{\lambda_t^1}{J_t}, \tilde{\lambda}_t^2 = \lambda_t^2, \tilde{\lambda}_t^3 = \lambda_t^3, \tilde{\lambda}_t^4 = \frac{\lambda_t^4}{U_{c,t}}, \tilde{\lambda}_t^5 = \frac{\lambda_t^5}{U_{c,t}b_t}, \tilde{\lambda}_t^6 = \lambda_t^6$, then the first-order conditions can be simplified as

$$Y_t: \; U_{Y,t} - \tilde{\lambda}_t^2(\varphi+1)A_t^{-(\varphi+1)}Y_t^{\varphi} - \tilde{\lambda}_t^3(1-\tau_t)(U_{c,t}+U_{cc,t}Y_t) - \tilde{\lambda}_t^4\tau_t U_{c,t} - \beta\sigma\tilde{\lambda}_t^5\frac{(1+\rho Q_{t+1})b_t}{\pi_{t+1}}C_{t+1}^{-\sigma}C_t^{-1} + \sigma\tilde{\lambda}_{t-1}^5\frac{(1+\rho Q_t)b_{t-1}}{\pi_t}C_t^{-\sigma-1} = 0 \tag{B.7}$$

$$\pi_t: \; \tilde{\lambda}_t^1 J_t\frac{1+\epsilon\varphi}{1-\epsilon}p(\pi_t)^{\frac{1+\epsilon\varphi}{1-\epsilon}-1}p'(\pi_t) - \tilde{\lambda}_{t-1}^2\theta\epsilon(1+\varphi)K_t\pi_t^{\epsilon(1+\varphi)-1} - \tilde{\lambda}_{t-1}^3\theta(\epsilon-1)J_t\pi_t^{\epsilon-2} - \tilde{\lambda}_t^4U_{c,t}(1+\rho Q_t^M)b_{t-1}\pi_t^{-2} + \tilde{\lambda}_{t-1}^5U_{c,t}(1+\rho Q_t^M)b_{t-1}\pi_t^{-2} - \tilde{\lambda}_t^6[(1-\theta)\frac{\epsilon(1+\varphi)}{\epsilon-1}p(\pi_t)^{\frac{1+\epsilon\varphi}{\epsilon-1}}p'(\pi_t) + \theta\epsilon(1+\varphi)\Delta_{t-1}\pi_t^{\epsilon(1+\varphi)-1}] = 0 \tag{B.8}$$

$$\Delta_t: \; U_{\Delta,t} + \tilde{\lambda}_t^6 - E_t\tilde{\lambda}_{t+1}^6\beta\theta\pi_{t+1}^{\epsilon(1+\varphi)} = 0 \tag{B.9}$$

$$K_t: \; -\frac{\epsilon}{\epsilon-1}\tilde{\lambda}_t^1 + \tilde{\lambda}_t^2 - \tilde{\lambda}_{t-1}^2\theta\pi_t^{\epsilon(1+\varphi)} = 0 \tag{B.10}$$

$$J_t: \; \tilde{\lambda}_t^1 + \tilde{\lambda}_t^3 - \tilde{\lambda}_{t-1}^3\theta\pi_t^{\epsilon-1} = 0 \tag{B.11}$$

$$\tau_t: \; \tilde{\lambda}_t^3 - \tilde{\lambda}_t^4 = 0 \tag{B.12}$$

$$b_t: \; \tilde{\lambda}_t^4 - E_t\tilde{\lambda}_{t+1}^4 = 0 \tag{B.13}$$

$$Q_t^M: \; \tilde{\lambda}_t^4(\rho\frac{b_{t-1}}{\pi_t} - b_t) + \tilde{\lambda}_t^5 b_t - \tilde{\lambda}_{t-1}^5\rho\frac{b_{t-1}}{\pi_t} = 0 \tag{B.14}$$

where $\tilde{\lambda}_t^1$ through $\tilde{\lambda}_t^6$ are the Lagrange multipliers. Condition (B.13) implies that the evolution of the Lagrange multiplier corresponding to the government budget $\tilde{\lambda}_t^4$ obeys a martingale. Condition (B.14) connects $\tilde{\lambda}_t^4$ to the Lagrange multiplier corresponding to the maturity structure $\tilde{\lambda}_t^5$. Conditions (B.10) – (B.12) relate $\tilde{\lambda}_t^4$ to the Lagrange multiplier corresponding to the aggregate supply relations $\tilde{\lambda}_t^1$, $\tilde{\lambda}_t^2$, $\tilde{\lambda}_t^3$. Conditions (B.7) and (B.9) implicitly determine the shadow price of the each constraint. Notice that (B.9) determines the marginal utility loss from inflation, while (B.7) determines the marginal utility loss from variations in output-gap.

B.1 Deterministic steady state Using the optimal allocation that appendix B describes, in a steady state with zero net inflation, $\bar{\pi} = 1$, we have

$$\bar{\Delta} = 1, \quad \frac{\bar{K}}{\bar{J}} = \frac{\epsilon-1}{\epsilon}, \quad \bar{Q}^s = \beta \quad \frac{\bar{b}}{\bar{S}} = \frac{1-\beta\rho}{1-\beta}$$

The associated steady-state price of long-term bond is given by

$$\bar{Q}^M = \frac{\beta}{1-\beta\rho}$$

which is increasing in average maturity ρ. The intuition is very straightforward, long-term debt yields more coupon payments and therefore demands higher price.

The steady-state government budget constraint implies

$$\bar{\tau} - s_g - s_z = \left(\beta^{-1} - 1\right) s_b$$

where $s_b \equiv \bar{Q}^M \bar{B}/\bar{Y}$ is the steady-state debt to GDP ratio, $s_g \equiv \bar{G}/\bar{Y}$ is the steady state government purchases to GDP ratio, $s_z \equiv \bar{Z}/\bar{Y}$ is the steady-state government transfers to GDP ratio.

Steady-state Lagrangian multipliers satisfy

$$\bar{\tilde{\lambda}}^1 = (\theta - 1)\bar{\tilde{\lambda}}^3 \tag{B.15}$$

$$\bar{\tilde{\lambda}}^2 = \frac{\epsilon}{1-\epsilon}\bar{\tilde{\lambda}}^3 \tag{B.16}$$

$$\bar{\tilde{\lambda}}^3 = \bar{\tilde{\lambda}}^4 \tag{B.17}$$

$$\bar{\tilde{\lambda}}^5 = \bar{\tilde{\lambda}}^4 \tag{B.18}$$

$$(1-\beta\theta)\bar{\tilde{\lambda}}^6 = -U_\Delta(\bar{Y}, 1) \tag{B.19}$$

$$[\frac{\epsilon(\varphi+1)}{1-\epsilon}\bar{Y}^{\varphi} + \bar{U}_c + (1-\bar{\tau})\bar{U}_{cc}\bar{Y} - \frac{(1-\beta)\sigma}{1-\beta\rho}\frac{s_b}{s_c}\bar{U}_c]\bar{\tilde{\lambda}}^4 = U_Y(\bar{Y}, 1) \tag{B.20}$$

Note that $\bar{\tilde{\lambda}}^4$ and $\bar{\tilde{\lambda}}^6$ can be solved from (B.19) and (B.20), and at steady state the other multipliers are proportional to $\bar{\tilde{\lambda}}^4$. At steady state, the Lagrange multiplier associated with government budget therefore completely summarizes the distortions from output; the price dispersion summarizes the distortions from inflation.

C Second-Order Approximation to Utility

The life-time welfare of household is defined by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, \xi_t) \tag{C.1}$$

where

$$\begin{aligned} U(Y_t, \Delta_t, \xi_t) &= \frac{(Y_t - G_t)^{1-\sigma}}{1-\sigma} - \frac{(\frac{Y_t}{A_t})^{1+\varphi}}{1+\varphi}\Delta_t \\ &= u(Y_t, G_t) - v(Y_t, \Delta_t, A_t) \end{aligned} \tag{C.2}$$

We use a second-order Taylor expansion for a variable X_t:

$$X_t/\bar{X} = e^{\ln X_t/\bar{X}} = e^{\hat{X}_t} = 1 + \hat{X}_t + \frac{1}{2}\hat{X}_t^2 \tag{C.3}$$

and

$$\tilde{X}_t = X_t - \bar{X} = \bar{X}(\hat{X}_t + \frac{1}{2}\hat{X}_t^2) \tag{C.4}$$

The derivation of second-order approximation closely follows Benigno and Woodford (2004).

The first term in (C.2) can be approximated to second order as

$$\begin{aligned}
u(Y_t, G_t) - \bar{u} &= \bar{u}_Y \tilde{Y}_t + \bar{u}_G \tilde{G}_t + \bar{u}_{YG} \tilde{Y}_t \tilde{G}_t + \frac{1}{2}\bar{u}_{YY}\tilde{Y}_t^{\,2} + \frac{1}{2}\bar{u}_{GG}\tilde{G}_t^{\,2} + \mathcal{O}(\|\xi_t\|^3) \\
&= \bar{u}_Y \bar{Y}(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^{\,2}) + \bar{u}_G \tilde{G}_t + \bar{u}_{YG}\bar{Y}\bar{G}\hat{Y}_t\hat{G}_t + \frac{1}{2}\bar{u}_{YY}\bar{Y}^2\hat{Y}_t^{\,2} + \frac{1}{2}\bar{u}_{GG}\tilde{G}_t^{\,2} + \mathcal{O}(\|\xi_t\|^3) \\
&= \bar{u}_Y \bar{Y}(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^{\,2}) + \bar{u}_{YG}\bar{Y}\bar{G}\hat{Y}_t\hat{G}_t + \frac{1}{2}\bar{u}_{YY}\bar{Y}^2\hat{Y}_t^{\,2} + \mathcal{O}(\|\xi_t\|^3) + t.i.p. \\
&= \bar{u}_Y \bar{Y}[\hat{Y}_t + \frac{1}{2}(1 + \frac{\bar{u}_{YY}}{\bar{u}_Y}\bar{Y})\hat{Y}_t^{\,2} - \frac{\bar{u}_{YY}}{\bar{u}_Y}\bar{G}\hat{G}_t\hat{Y}_t] + \mathcal{O}(\|\xi_t\|^3) + t.i.p. \\
&= \bar{u}_Y \bar{Y}[\hat{Y}_t + \frac{1}{2}(1 - s_c^{-1}\sigma)\hat{Y}_t^{\,2} + s_g s_c^{-1}\sigma\hat{G}_t\hat{Y}_t] + \mathcal{O}(\|\xi_t\|^3) + t.i.p.
\end{aligned} \tag{C.5}$$

"t.i.p." represents the terms that are independent of policy. The second term in (C.2) can be approximated by

$$\begin{aligned}
v(Y_t, \Delta_t, A_t) - \bar{v} =&\bar{v}_Y\tilde{Y}_t + \bar{v}_A\tilde{A}_t + \bar{v}_\Delta\tilde{\Delta}_t + \bar{v}_{YA}\tilde{Y}_t\tilde{A}_t + \bar{v}_{Y\Delta}\tilde{Y}_t\tilde{\Delta}_t + \bar{v}_{\Delta A}\tilde{\Delta}_t\tilde{A}_t \\
&+ \frac{1}{2}\bar{v}_{YY}\tilde{Y}_t^{\,2} + \frac{1}{2}\bar{v}_{AA}\tilde{A}_t^2 + \frac{1}{2}\bar{v}_{\Delta\Delta}\tilde{\Delta}_t^2 + \mathcal{O}(\|\xi_t\|^3) \\
=&\bar{v}_Y\tilde{Y}_t + \bar{v}_\Delta\tilde{\Delta}_t + \bar{v}_{YA}\tilde{Y}_t\tilde{A}_t + \bar{v}_{Y\Delta}\tilde{Y}_t\tilde{\Delta}_t + \bar{v}_{\Delta A}\tilde{\Delta}_t\tilde{A}_t \\
&+ \frac{1}{2}\bar{v}_{YY}\tilde{Y}_t^{\,2} + \frac{1}{2}\bar{v}_{\Delta\Delta}\tilde{\Delta}_t^2 + \mathcal{O}(\|\xi_t\|^3) + t.i.p. \\
=&\bar{v}_Y\bar{Y}(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^{\,2}) + \bar{v}_\Delta\tilde{\Delta}_t + \bar{v}_{YA}\bar{Y}\hat{Y}_t\tilde{A}_t + \bar{v}_{Y\Delta}\bar{Y}\hat{Y}_t\tilde{\Delta}_t + \bar{v}_{\Delta A}\tilde{\Delta}_t\tilde{A}_t \\
&+ \frac{1}{2}\bar{v}_{YY}\bar{Y}^2\hat{Y}_t^{\,2} + \frac{1}{2}\bar{v}_{\Delta\Delta}\tilde{\Delta}_t^2 + \mathcal{O}(\|\xi_t\|^3) + t.i.p. \\
=&\bar{v}_Y\bar{Y}[\hat{Y}_t + \frac{1}{2}(1 + \frac{\bar{v}_{YY}\bar{Y}}{\bar{v}_Y})\hat{Y}_t^{\,2} + \frac{\bar{v}_\Delta}{\bar{v}_Y\bar{Y}}\tilde{\Delta}_t + \frac{\bar{v}_{Y\Delta}}{\bar{v}_Y}\tilde{\Delta}_t\hat{Y}_t + \frac{\bar{v}_{YA}}{\bar{v}_Y}\tilde{A}_t\hat{Y}_t + \frac{\bar{v}_{\Delta A}}{\bar{v}_Y\bar{Y}}\tilde{\Delta}_t\tilde{A}_t + \frac{1}{2}\frac{\bar{v}_{\Delta\Delta}}{\bar{v}_Y\bar{Y}}\tilde{\Delta}_t^2] \\
&+ \mathcal{O}(\|\xi_t\|^3) + t.i.p. \\
=&\bar{v}_Y\bar{Y}[\hat{Y}_t + \frac{1}{2}(1+\varphi)\hat{Y}_t^{\,2} + \frac{1}{1+\varphi}\tilde{\Delta}_t + \tilde{\Delta}_t\hat{Y}_t - (1+\varphi)\hat{Y}_t\tilde{A}_t - \tilde{\Delta}_t\tilde{A}_t] + \mathcal{O}(\|\xi_t\|^3) + t.i.p.
\end{aligned} \tag{C.6}$$

From Benigno and Woodford (2004) we know that a second order approximation to (B.6) yields

$$\tilde{\Delta}_t = \theta\tilde{\Delta}_{t-1} + \frac{\theta\epsilon}{1-\theta}(1+\varphi)(1+\epsilon\varphi)\frac{\hat{\pi}_t^2}{2} + \mathcal{O}(\|\xi_t\|^3) + t.i.p. \tag{C.7}$$

which implies that $\tilde{\Delta}_t = \mathcal{O}(\pi_t^2)$.

Therefore (C.6) can be simplified as

$$v(Y_t,\Delta_t,A_t)-\bar{v}=\bar{v}_Y\bar{Y}[\hat{Y}_t+\frac{1}{2}(1+\varphi)\hat{Y}_t^{\ 2}+\frac{1}{1+\varphi}\tilde{\Delta}_t-(1+\varphi)\hat{Y}_t\hat{A}_t]+\mathcal{O}(\|\xi_t\|^3)+t.i.p. \quad \text{(C.8)}$$

Combine (C.5) and (C.8) and apply the relation $\bar{v}_Y=(1-\Phi)\bar{u}_Y$, we approximate the life-time utility (C.1) as

$$\begin{aligned}U_0-\bar{U}_0=&\bar{u}_Y\bar{Y}E_0\sum_{t=0}^{\infty}\beta^t\{\Phi\hat{Y}_t+\frac{1}{2}[(1-\frac{\sigma}{s_c})-(1-\Phi)(1+\varphi)]\hat{Y}_t^2+[s_g{s_c}^{-1}\sigma\hat{G}_t+(1-\Phi)(1+\varphi)\hat{A}_t]\hat{Y}_t-\frac{1-\Phi}{1+\varphi}\hat{\Delta}_t\}\\&+\mathcal{O}(\|\xi_t\|^3)+t.i.p. \quad \text{(C.9)}\end{aligned}$$

From Benigno and Woodford (2004) we observe

$$E_0\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t=\frac{\theta\epsilon}{(1-\theta)(1-\beta\theta)}(1+\varphi)(1+\epsilon\varphi)\sum_{t=0}^{\infty}\beta^t\frac{\hat{\pi}_t^2}{2} \quad \text{(C.10)}$$

Therefore, the second-order approximation to the life-time utility (C.1) can be further expressed as

$$U_0-\bar{U}_0=\bar{u}_Y\bar{Y}E_0\sum_{t=0}^{\infty}\beta^t\{A_y\hat{Y}_t+\frac{1}{2}A_{yy}\hat{Y}_t^2+A_\xi'\hat{\xi}_t\hat{Y}_t-\frac{A_\pi}{2}\hat{\pi}_t^2\}+\mathcal{O}(\|\xi_t\|^3)+t.i.p. \quad \text{(C.11)}$$

where

$$\begin{aligned}A_y&=\Phi\\A_{yy}&=(1-\sigma{s_c}^{-1})-(1-\Phi)(1+\varphi)\\A_\xi'\hat{\xi}_t&=\sigma{s_c}^{-1}s_g\hat{G}_t+(1-\Phi)(1+\varphi)\hat{A}_t\\A_\pi&=(1-\Phi)\frac{\theta\epsilon(1+\epsilon\varphi)}{(1-\theta)(1-\beta\theta)}\end{aligned}$$

$\Phi=1-(1-\tau)\frac{\epsilon-1}{\epsilon}$ measures the inefficiency of steady state of output. $s_c=\frac{\bar{C}}{\bar{Y}}$ is steady state consumption to GDP ratio; $s_g=\frac{\bar{G}}{\bar{Y}}$ is steady state government spending to GDP ratio.

D Second-Order Approximation to Government's IEC

Recall the government budget constraint

$$Q_t^Mb_t^M+S_t=(1+\rho Q_t^M)\frac{b_{t-1}^M}{\pi_t} \quad \text{(D.1)}$$

and no-arbitrage condition

$$Q_t^M = \beta E_t \frac{U_{c,t+1}}{U_{c,t}} \frac{1}{\pi_{t+1}} (1 + \rho Q_{t+1}^M) \tag{D.2}$$

Define

$$W_t = U_{c,t}(1 + \rho Q_t^M) \frac{b_{t-1}^M}{\pi_t} \tag{D.3}$$

By applying (D.2) and (D.3), (D.1) can be rewritten as

$$W_t = U_{c,t} S_t + \beta E_t W_{t+1} \tag{D.4}$$

and

$$W_t = E_t \sum_{k=0}^{\infty} U_{c,t+k} S_{t+k} \tag{D.5}$$

A second-order approximation to $U_{c,t}S_t$ yields

$$U_{c,t}S_t = \bar{U}_c \bar{S} + \bar{U}_{cc}\bar{S}\tilde{C}_t + \bar{U}_c \tilde{S}_t + \frac{1}{2}\bar{S}\bar{U}_{ccc}\tilde{C}_t^2 + \bar{U}_{cc}\tilde{S}_t\tilde{C}_t \tag{D.6}$$

We express $\tilde{C}_t$ in terms of $\hat{Y}_t$ and $\hat{G}_t$ through the second order approximation to the identity $C_t = Y_t - G_t$,

$$\tilde{C}_t = \bar{Y}(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2) - \bar{G}(\hat{G}_t + \frac{1}{2}\hat{G}_t^2) \tag{D.7}$$

Since $S_t = \tau_t Y_t - G_t - Z_t$, a second-order approximation to the primary surplus can be written as

$$\tilde{S}_t = \bar{\tau}\bar{Y}(\hat{\tau}_t + \hat{Y}_t) + \frac{1}{2}\bar{\tau}\bar{Y}(\hat{\tau}_t^2 + \hat{Y}_t^2) + \bar{\tau}\bar{Y}\hat{Y}_t\hat{\tau}_t - \bar{G}(\hat{G}_t + \frac{1}{2}\hat{G}_t^2) - \bar{Z}(\hat{Z}_t + \frac{1}{2}\hat{Z}_t^2) \tag{D.8}$$

Substituting (D.7) and (D.8) into (D.6), we obtain

$$
\begin{aligned}
U_{c,t}S_t - \bar{U}_c\bar{S} =& \bar{U}_{cc}\bar{S}[\bar{Y}(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2) - \bar{G}(\hat{G}_t + \frac{1}{2}\hat{G}_t^2)] \\
& + \bar{U}_c[\bar{\tau}\bar{Y}(\hat{\tau}_t + \hat{Y}_t) + \frac{1}{2}\bar{\tau}\bar{Y}(\hat{\tau}_t^2 + \hat{Y}_t^2) + \bar{\tau}\bar{Y}\hat{Y}_t\hat{\tau}_t - \bar{G}(\hat{G}_t + \frac{1}{2}\hat{G}_t^2) - \bar{Z}(\hat{Z}_t + \frac{1}{2}\hat{Z}_t^2)] \\
& + \frac{1}{2}\bar{S}\bar{U}_{ccc}(\bar{Y}^2\hat{Y}_t^2 + \bar{G}^2\hat{G}_t^2 - 2\bar{Y}\bar{G}\hat{Y}_t\hat{G}_t) \\
& + \bar{U}_{cc}[\bar{\tau}\bar{Y}^2\hat{Y}_t^2 + \bar{\tau}\bar{Y}^2\hat{Y}_t\hat{\tau}_t - (\bar{\tau}+1)\bar{Y}\bar{G}\hat{Y}_t\hat{G}_t - \bar{Y}\bar{Z}\hat{Y}_t\hat{Z}_t - \bar{\tau}\bar{Y}\bar{G}\hat{\tau}_t\hat{G}_t + \bar{G}^2\hat{G}_t^2 + \bar{G}\bar{Z}\hat{G}_t\hat{Z}_t] \\
& + \mathcal{O}(\|\xi_t\|^3) \\
=& \bar{U}_c\bar{S}[-\sigma s_c^{-1}(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2) + \sigma s_c^{-1}s_g(\hat{G}_t + \frac{1}{2}\hat{G}_t^2) + \frac{\bar{\tau}\bar{Y}}{\bar{S}}(\hat{\tau}_t + \hat{Y}_t) + \frac{1}{2}\frac{\bar{\tau}\bar{Y}}{\bar{S}}(\hat{\tau}_t^2 + \hat{Y}_t^2) + \frac{\bar{\tau}\bar{Y}}{\bar{S}}\hat{Y}_t\hat{\tau}_t - \frac{\bar{G}}{\bar{S}}\hat{G}_t - \frac{\bar{Z}}{\bar{S}}\hat{Z}_t \\
& + \frac{1}{2}\sigma(1+\sigma)(s_c^{-2}\hat{Y}_t^2 + s_g^2 s_c^{-2}\hat{G}_t^2 - 2s_g s_c^{-2}\hat{Y}_t\hat{G}_t) \\
& - \sigma s_c^{-1}\frac{\bar{\tau}\bar{Y}}{\bar{S}}(\hat{Y}_t^2 + \hat{Y}_t\hat{\tau}_t) + \sigma s_c^{-1}s_g(\frac{\bar{\tau}\bar{Y}}{\bar{S}} + \frac{\bar{Y}}{\bar{S}})\hat{Y}_t\hat{G}_t + \sigma s_c^{-1}s_z\frac{\bar{Y}}{\bar{S}}\hat{Y}_t\hat{Z}_t + \sigma s_c^{-1}s_g\frac{\bar{\tau}\bar{Y}}{\bar{S}}\hat{\tau}_t\hat{G}_t] \\
& + \mathcal{O}(\|\xi_t\|^3) \\
=& \bar{U}_c\bar{S}\{(-\frac{\sigma}{s_c} + \frac{\bar{\tau}\bar{Y}}{\bar{S}})\hat{Y}_t + \frac{\bar{\tau}\bar{Y}}{\bar{S}}\hat{\tau}_t + \frac{1}{2}\frac{\bar{\tau}\bar{Y}}{\bar{S}}\hat{\tau}_t^2 + \frac{1}{2}[\frac{\bar{\tau}\bar{Y}}{\bar{S}}(1 - \frac{2\sigma}{s_c}) - \frac{\sigma}{s_c} + \frac{\sigma^2}{s_c^2} + \frac{\sigma}{s_c^2}]\hat{Y}_t^2 + \frac{\bar{\tau}\bar{Y}}{\bar{S}}(1 - \sigma s_c^{-1})\hat{Y}_t\hat{\tau}_t \\
& + \sigma\frac{s_g}{s_c}\frac{\bar{\tau}\bar{Y}}{\bar{S}}\hat{G}_t\hat{\tau}_t + \sigma\frac{s_z}{s_c}\frac{\bar{Y}}{\bar{S}}\hat{Y}_t\hat{Z}_t - \sigma\frac{s_g}{s_c}(\frac{\sigma+1}{s_c} - \frac{\bar{\tau}\bar{Y}}{\bar{S}} - \frac{\bar{Y}}{\bar{S}})\hat{Y}_t\hat{G}_t - \frac{\bar{Z}}{\bar{S}}\hat{Z}_t + (\sigma s_c^{-1}s_g - \frac{\bar{G}}{\bar{S}})\hat{G}_t\} + \mathcal{O}(\|\xi_t\|^3)
\end{aligned}
\tag{D.9}
$$

Therefore, by substituting (D.9) into (D.5), we express the second-order approximation to government lEC as

$$
\begin{aligned}
\frac{W_0 - \bar{W}}{\bar{W}} =& (1-\beta)E_0\sum_{t=0}^{\infty}\beta^t[\{B_y\hat{Y}_t + B_\tau\hat{\tau}_t + B_{y\tau}\hat{Y}_t\hat{\tau}_t + \frac{1}{2}B_{yy}\hat{Y}_t^2 + \frac{1}{2}B_{\tau\tau}\hat{\tau}_t^2 + B'_{\xi_y}\hat{\xi}_t\hat{Y}_t + B'_{\xi_\tau}\hat{\xi}_t\hat{\tau}_t + B_\xi{}'\hat{\xi}_t\} \\
& + \mathcal{O}(\|\xi_t\|^3) + t.i.p.
\end{aligned}
\tag{D.10}
$$

where

$$
B_y = -\frac{\sigma}{s_c} + \frac{\bar{\tau}\bar{Y}}{\bar{S}} \qquad B_\tau = \frac{\bar{\tau}\bar{Y}}{\bar{S}} \qquad B_\xi{}'\hat{\xi}_t = -\frac{s_z}{s_d}\hat{Z}_t + (\sigma\frac{s_c}{s_g} - \frac{s_g}{s_d})\hat{G}_t
$$

$$
B_{y\tau} = \frac{\bar{\tau}\bar{Y}}{\bar{S}}(1 - \sigma s_c^{-1}) \qquad B_{\tau\tau} = \frac{\bar{\tau}\bar{Y}}{\bar{S}} \qquad B_{yy} = \frac{\bar{\tau}\bar{Y}}{\bar{S}}(1 - \frac{2\sigma}{s_c}) - \frac{\sigma}{s_c} + \frac{\sigma^2}{s_c^2} + \frac{\sigma}{s_c^2}
$$

$$
B'_{\xi_y}\hat{\xi}_t = \sigma\frac{s_z}{s_c}\frac{1}{s_d}\hat{Z}_t - \sigma\frac{s_g}{s_c}(\frac{\sigma+1}{s_c} - \frac{\bar{\tau}\bar{Y}}{\bar{S}} - \frac{1}{s_d})\hat{G}_t \qquad B'_{\xi_\tau}\hat{\xi}_t = \sigma\frac{s_g}{s_c}\frac{\bar{\tau}\bar{Y}}{\bar{S}}\hat{G}_t
$$

$s_z = \frac{\bar{Z}}{\bar{Y}}$ is steady state government transfer payment to GDP ratio; $s_d = \frac{\bar{S}}{\bar{Y}}$ is steady state surplus to GDP ratio.

E Second-Order Approximation to Aggregate Supply Relation

The aggregate supply relation is defined by the equations

$$J_t[\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}]^{\frac{1+\epsilon\varphi}{1-\epsilon}} = \frac{\epsilon}{\epsilon-1}K_t \tag{E.1}$$

$$K_t = \mu_t^W(\frac{Y_t}{A_t})^{\varphi+1} + \beta\theta E_t K_{t+1}\pi_{t+1}^{\epsilon(1+\varphi)} \tag{E.2}$$

$$J_t = (1-\tau_t)U_{c,t}Y_t + \beta\theta E_t J_{t+1}\pi_{t+1}^{\epsilon-1} \tag{E.3}$$

A second-order approximation to (E.1) can be written as

$$\frac{\epsilon}{\epsilon-1}\tilde{K}_t - \tilde{J}_t = \bar{J}\frac{\theta}{1-\theta}(1+\epsilon\varphi)\{\tilde{\pi}_t + \frac{1}{2}[\frac{\theta}{1-\theta}\epsilon(\varphi+1)+(\epsilon-2)]\tilde{\pi}_t^2\} + \frac{\theta}{1-\theta}(1+\epsilon\varphi)\tilde{J}_t\tilde{\pi}_t + \mathcal{O}(\|\xi_t\|^3) \tag{E.4}$$

A second-order approximation to (E.2) can be written as

$$\begin{aligned}\tilde{K}_t =& \beta\theta E_t\tilde{K}_{t+1} + \beta\theta\epsilon(1+\varphi)\bar{K}\{E_t\tilde{\pi}_{t+1} + \frac{1}{2}[\epsilon(1+\varphi)-1]E_t\tilde{\pi}_{t+1}^2\} + \beta\theta\epsilon(1+\varphi)\tilde{K}_{t+1}\tilde{\pi}_{t+1}\\ &+ \bar{\mu}^W(1+\varphi)\bar{Y}^{\varphi}\tilde{Y}_t - \bar{\mu}^W(1+\varphi)^2\bar{Y}^{\varphi}\tilde{Y}_t\tilde{A}_t + (\varphi+1)\bar{Y}^{\varphi}\tilde{\mu}_t^W\tilde{Y}_t + \frac{1}{2}\bar{\mu}^W\varphi(1+\varphi)\bar{Y}^{\varphi-1}\tilde{Y}_t^2\\ &- \bar{\mu}^W(\varphi+1)\bar{Y}^{\varphi+1}\tilde{A}_t + \bar{Y}^{\varphi+1}\tilde{\mu}_t^W + \mathcal{O}(\|\xi_t\|^3) + t.i.p.\end{aligned} \tag{E.5}$$

A second-order approximation to (E.3) can be written as

$$\begin{aligned}\tilde{J}_t =& \beta\theta E_t\tilde{J}_{t+1} + \beta\theta(\epsilon-1)\bar{J}[E_t\tilde{\pi}_{t+1} + \frac{1}{2}(\epsilon-2)E_t\tilde{\pi}_{t+1}^2] + \beta\theta(\epsilon-1)\tilde{J}_{t+1}\tilde{\pi}_{t+1}\\ &+ (1-\bar{\tau})(\bar{U}_c + \bar{U}_{cc}\bar{Y})\tilde{Y}_t - \bar{U}_c\bar{Y}\tilde{\tau}_t + \frac{1}{2}(1-\bar{\tau})(2\bar{U}_{cc} + \bar{U}_{ccc}\bar{Y})\tilde{Y}_t^2 - (\bar{U}_c + \bar{U}_{cc}\bar{Y})\tilde{Y}_t\tilde{\tau}_t\\ &+ \bar{U}_{cc}\bar{Y}\tilde{\tau}_t\tilde{G}_t - (1-\bar{\tau})(\bar{U}_{cc} + \bar{U}_{ccc}\bar{Y})\tilde{Y}_t\tilde{G}_t - (1-\bar{\tau})\bar{Y}\bar{U}_{cc}\tilde{G}_t + \mathcal{O}(\|\xi_t\|^3) + t.i.p.\end{aligned} \tag{E.6}$$

Therefore, $\frac{\epsilon}{\epsilon-1}$(E.5)-(E.6) can be expressed as

$$\begin{aligned}\frac{\epsilon}{\epsilon-1}\tilde{K}_t - \tilde{J}_t =& \beta\theta E_t(\frac{\epsilon}{\epsilon-1}\tilde{K}_{t+1} - \tilde{J}_{t+1})\\ &+\frac{\epsilon}{\epsilon-1}\beta\theta\epsilon(1+\varphi)\bar{K}\{E_t\tilde{\pi}_{t+1} + \frac{1}{2}[\epsilon(1+\varphi)-1]E_t\tilde{\pi}_{t+1}^2\} - \beta\theta(\epsilon-1)\bar{J}[E_t\tilde{\pi}_{t+1} + \frac{1}{2}(\epsilon-2)E_t\tilde{\pi}_{t+1}^2]\\ &+\frac{\epsilon}{\epsilon-1}\beta\theta\epsilon(1+\varphi)\tilde{K}_{t+1}\tilde{\pi}_{t+1} - \beta\theta(\epsilon-1)\tilde{J}_{t+1}\tilde{\pi}_{t+1}\\ &+\frac{\epsilon}{\epsilon-1}[\bar{\mu}^W(1+\varphi)\bar{Y}^{\varphi}\tilde{Y}_t - \bar{\mu}^W(1+\varphi)^2\bar{Y}^{\varphi}\tilde{Y}_t\tilde{A}_t + (\varphi+1)\bar{Y}^{\varphi}\tilde{\mu}_t^W\tilde{Y}_t + \frac{1}{2}\bar{\mu}^W\varphi(1+\varphi)\bar{Y}^{\varphi-1}\tilde{Y}_t^2\\ &\quad - \bar{\mu}^W(\varphi+1)\bar{Y}^{\varphi+1}\tilde{A}_t + \bar{Y}^{\varphi+1}\tilde{\mu}_t^W]\\ &-[(1-\bar{\tau})(\bar{U}_c + \bar{U}_{cc}\bar{Y})\tilde{Y}_t - \bar{U}_c\bar{Y}\tilde{\tau}_t + \frac{1}{2}(1-\bar{\tau})(2\bar{U}_{cc} + \bar{U}_{ccc}\bar{Y})\tilde{Y}_t^2 - (\bar{U}_c + \bar{U}_{cc}\bar{Y})\tilde{Y}_t\tilde{\tau}_t]\\ &-[\bar{U}_{cc}\bar{Y}\tilde{\tau}_t\tilde{G}_t - (1-\bar{\tau})(\bar{U}_{cc} + \bar{U}_{ccc}\bar{Y})\tilde{Y}_t\tilde{G}_t - (1-\bar{\tau})\bar{Y}\bar{U}_{cc}\tilde{G}_t] + \mathcal{O}(\|\xi_t\|^3) + t.i.p.\end{aligned} \tag{E.7}$$

Then we plug (E.4) into (E.7) and obtain

$$\begin{aligned}
&\bar{J}\frac{\theta}{1-\theta}(1+\epsilon\varphi)\{\tilde{\pi}_t + \frac{1}{2}[\frac{\theta}{1-\theta}\epsilon(\varphi+1)+(\epsilon-2)]\tilde{\pi}_t^2\} + \frac{\theta}{1-\theta}(1+\epsilon\varphi)\tilde{J}_t\tilde{\pi}_t \\
=&\beta\theta\bar{J}\frac{\theta}{1-\theta}(1+\epsilon\varphi)\{E_t\tilde{\pi}_{t+1} + \frac{1}{2}[\frac{\theta}{1-\theta}\epsilon(\varphi+1)+(\epsilon-2)]E_t\tilde{\pi}_{t+1}^2\} + \beta\theta\frac{\theta}{1-\theta}(1+\epsilon\varphi)\tilde{J}_{t+1}\tilde{\pi}_{t+1} \\
&+\frac{\epsilon}{\epsilon-1}\beta\theta\epsilon(1+\varphi)\bar{K}\{E_t\tilde{\pi}_{t+1} + \frac{1}{2}[\epsilon(1+\varphi)-1]E_t\tilde{\pi}_{t+1}^2\} - \beta\theta(\epsilon-1)\bar{J}[E_t\tilde{\pi}_{t+1} + \frac{1}{2}(\epsilon-2)E_t\tilde{\pi}_{t+1}^2] \\
&+\beta\theta\epsilon(1+\varphi)[\tilde{J}_{t+1}\tilde{\pi}_{t+1} + \bar{J}\frac{\theta}{1-\theta}(1+\epsilon\varphi)\tilde{\pi}_{t+1}^2] - \beta\theta(\epsilon-1)\tilde{J}_{t+1}\tilde{\pi}_{t+1} \\
&+\frac{\epsilon}{\epsilon-1}[\bar{\mu}^W(1+\varphi)\bar{Y}^{\varphi}\tilde{Y}_t - \bar{\mu}^W(1+\varphi)^2\bar{Y}^{\varphi}\tilde{Y}_t\tilde{A}_t + (\varphi+1)\bar{Y}^{\varphi}\tilde{\mu}_t^W\tilde{Y}_t + \frac{1}{2}\bar{\mu}^W\varphi(1+\varphi)\bar{Y}^{\varphi-1}\tilde{Y}_t^2 \\
&-\bar{\mu}^W(\varphi+1)\bar{Y}^{\varphi+1}\tilde{A}_t + \bar{Y}^{\varphi+1}\tilde{\mu}_t^W] \\
&-[(1-\bar{\tau})(\bar{U}_c+\bar{U}_{cc}\bar{Y})\tilde{Y}_t - \bar{U}_c\bar{Y}\tilde{\tau}_t + \frac{1}{2}(1-\bar{\tau})(2\bar{U}_{cc}+\bar{U}_{ccc}\bar{Y})\tilde{Y}_t^2 - (\bar{U}_c+\bar{U}_{cc}\bar{Y})\tilde{Y}_t\tilde{\tau}_t] \\
&-[\bar{U}_{cc}\bar{Y}\tilde{\tau}_t\tilde{G}_t - (1-\bar{\tau})(\bar{U}_{cc}+\bar{U}_{ccc}\bar{Y})\tilde{Y}_t\tilde{G}_t - (1-\bar{\tau})\bar{Y}\bar{U}_{cc}\tilde{G}_t] + \mathcal{O}(\|\xi_t\|^3) + t.i.p. \qquad \text{(E.8)}
\end{aligned}$$

Note that at steady state we have the relations $\frac{\bar{K}}{\bar{J}} = \frac{\epsilon-1}{\epsilon}$, $(1-\beta\theta)\bar{K} = \bar{Y}^{\varphi+1}$ and $(1-\beta\theta)\bar{J} = (1-\bar{\tau})\bar{U}_c\bar{Y}$, therefore (E.8) can be simplified as

$$\begin{aligned}
&\frac{\theta}{1-\theta}(1+\epsilon\varphi)\hat{\pi}_t + \frac{1}{2}\frac{\theta}{1-\theta}(1+\epsilon\varphi)[\frac{\theta}{1-\theta}\epsilon(\varphi+1)+(\epsilon-1)]\hat{\pi}_t^2 + \frac{\theta}{1-\theta}(1+\epsilon\varphi)\bar{J}^{-1}\hat{J}_t\hat{\pi}_t \\
=&\frac{\theta}{1-\theta}(1+\epsilon\varphi)\beta E_t\hat{\pi}_{t+1} + \frac{1}{2}\frac{\theta}{1-\theta}(1+\epsilon\varphi)\beta[\frac{1}{1-\theta}\epsilon(\varphi+1)+(\epsilon-1)]\hat{\pi}_{t+1}^2 + \beta\frac{\theta}{1-\theta}(1+\epsilon\varphi)\bar{J}^{-1}\hat{J}_{t+1}\hat{\pi}_{t+1} \\
&+(1-\beta\theta)[(1+\varphi)\hat{Y}_t - (1+\varphi)^2\hat{Y}_t\hat{A}_t + (1+\varphi)\hat{Y}_t\hat{\mu}_t^W + \frac{1}{2}(1+\varphi)^2\hat{Y}_t^2 - (1+\varphi)\hat{A}_t + \hat{\mu}_t^W] \\
&-(1-\beta\theta)\{(1-\sigma s_c^{-1})\hat{Y}_t - w_\tau\hat{\tau}_t - w_\tau(1-\sigma s_c^{-1})\hat{Y}_t\hat{\tau}_t - \frac{1}{2}w_\tau\hat{\tau}_t^2 + \frac{1}{2}[1-3\sigma s_c^{-1}+\sigma(1+\sigma)s_c^{-2}]\hat{Y}_t^2 \\
&-\sigma\frac{s_g}{s_c}w_\tau\hat{\tau}_t\hat{G}_t - [\sigma(1+\sigma)s_c^{-2} - \sigma s_c^{-1}]s_g\hat{Y}_t\hat{G}_t + \sigma\frac{s_g}{s_c}\hat{G}_t\} + \mathcal{O}(\|\xi_t\|^3) + t.i.p. \qquad \text{(E.9)}
\end{aligned}$$

Define $V_t = \hat{\pi}_t + \frac{1}{2}[\frac{1}{1-\theta}\epsilon(\varphi+1)+(\epsilon-1)]\hat{\pi}_t^2 + \bar{J}^{-1}\hat{J}_t\hat{\pi}_t$, and substitue into (E.9), we obtain a recursive relation

$$\begin{aligned}
V_t =&\kappa\{C_y\hat{Y}_t + C_\tau\hat{\tau}_t + C_{y\tau}\hat{Y}_t\hat{\tau}_t + \frac{1}{2}C_{yy}\hat{Y}_t^2 + \frac{1}{2}C_{\tau\tau}\hat{\tau}_t^2 + C'_{\xi_y}\hat{\xi}_t\hat{Y}_t + C'_{\xi_\tau}\hat{\xi}_t\hat{\tau}_t + \frac{C_\pi}{2}\pi_t^2 + C_\xi{}'\hat{\xi}_t\}\} + \beta E_tV_{t+1} \\
&+\mathcal{O}(\|\xi_t\|^3) + t.i.p. \qquad \text{(E.10)}
\end{aligned}$$

where

$$C_y = 1 \qquad C_\tau = \psi \qquad C_\pi = \frac{\epsilon(1+\varphi)}{\kappa}$$
$$C_{y\tau} = (1-\sigma s_c^{-1})\psi \qquad C_{\tau\tau} = \psi \qquad C_{yy} = (2+\varphi-\sigma s_c^{-1}) + \sigma(s_c^{-1}-s_c^{-2})(\varphi+\sigma s_c^{-1})^{-1}$$
$$C'_{\xi_y}\hat{\xi}_t = \frac{\sigma^2 s_c^{-2} + \sigma s_c^{-2} - \sigma s_c^{-1}}{\varphi+\sigma s_c^{-1}} s_g \hat{G}_t - \frac{(1+\varphi)^2}{\varphi+\sigma s_c^{-1}}\hat{A}_t + \frac{1+\varphi}{\varphi+\sigma s_c^{-1}}\hat{\mu}_t^W \qquad C'_{\xi_\tau}\hat{\xi}_t = \sigma s_c{}^{-1} s_g \psi \hat{G}_t$$
$$C_\xi{}'\hat{\xi}_t = -\frac{\varphi+1}{\varphi+\sigma s_c^{-1}}\hat{A}_t - \frac{s_g}{s_c}\frac{\sigma}{\varphi+\sigma s_c^{-1}}\hat{G}_t$$

and

$$\kappa = \frac{1-\theta}{\theta}\frac{(1-\beta\theta)(\varphi+\sigma s_c^{-1})}{1+\epsilon\varphi}$$
$$w_\tau = \frac{\bar{\tau}}{1-\bar{\tau}}$$
$$\psi = \frac{w_\tau}{\varphi+\sigma s_c^{-1}}$$

Integrate (E.10) forward from $t = 0$, we have

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \kappa \{C_y \hat{Y}_t + C_\tau \hat{\tau}_t + C_{y\tau}\hat{Y}_t\hat{\tau}_t + \frac{1}{2}C_{yy}\hat{Y}_t^2 + \frac{1}{2}C_{\tau\tau}\hat{\tau}_t^2 + C'_{\xi_y}\hat{\xi}_t\hat{Y}_t + C'_{\xi_\tau}\hat{\xi}_t\hat{\tau}_t + \frac{C_\pi}{2}\hat{\pi}_t^2 + C_\xi{}'\hat{\xi}_t\}$$
$$+ \mathcal{O}(\|\xi_t\|^3) + t.i.p. \tag{E.11}$$

F Quadratic Approximation to Objective Function

Now we use a linear combination of (D.10) and (E.11) to eliminate the linear term in the second order approximation to the welfare measure. The coefficients μ_B, μ_C should satisfy

$$\mu_B B_y + \mu_C C_y = -\Phi$$
$$\mu_B B_\tau + \mu_C C_\tau = 0$$

The solution is

$$\mu_B = \frac{\Phi w_\tau}{\Gamma}$$
$$\mu_C = -\frac{\Phi(1+w_g)(\varphi+\sigma s_c^{-1})}{\Gamma}$$

where $w_g = \frac{\bar{G}+\bar{Z}}{\bar{S}}$ is steady-state government outlays to surplus ratio, and satisfies $1+w_g = \frac{\bar{\tau}\bar{Y}}{\bar{S}}$. $\Gamma = \sigma s_c^{-1} w_\tau + (1+w_g)(\varphi+\sigma s_c^{-1} - w_\tau)$.

Therefore, we can finally express the objective function in the linear quadratic form of

$$(\bar{u}_Y \bar{Y})^{-1}(U_0 - \bar{U}_0) + \mu_B(1-\beta)^{-1}\frac{W_0 - \bar{W}}{\bar{W}} + \mu_C \kappa^{-1} V_0 = -E_0 \sum_{t=0}^{\infty} \beta^t [\frac{1}{2} q_Y (\hat{Y}_t - \hat{Y}_t^e)^2 + \frac{1}{2} q_\pi \hat{\pi}_t^2] + \mathcal{O}(\|\xi_t\|^3) + t.i.p. \tag{F.1}$$

with

$$q_Y = (1-\Phi)(\varphi + \sigma s_c^{-1}) + \Phi(\varphi + \sigma s_c^{-1})\frac{(1+w_g)(1+\varphi)}{\Gamma} + \Phi\sigma s_c^{-1}\frac{(1+w_g)(1+w_\tau)}{\Gamma} - \Phi\sigma s_c^{-2}\frac{(1+w_g+w_\tau)}{\Gamma}$$
$$q_\pi = \frac{\Phi(1+w_g)\epsilon(1+\varphi)(\varphi+\sigma s_c^{-1})}{\kappa\Gamma} + \frac{(1-\Phi)\epsilon(\varphi+\sigma s_c^{-1})}{\kappa}$$

and $\hat{Y}_t^e$ denotes the efficient level of output, which is exogenous and depends on the vector of exogenous shocks ξ_t,

$$\begin{aligned}\hat{Y}_t^e =& q_Y^{-1}(A'_\xi \hat{\xi}_t + \mu_B B'_{\xi_y}\hat{\xi}_t + \mu_C C'_{\xi_y}\hat{\xi}_t) \\ =& q_A \hat{A}_t + q_G \hat{G}_t + q_Z \hat{Z}_t + q_W \hat{\mu}_t^W\end{aligned}$$

where

$$q_A = q_Y^{-1}[(1-\Phi)(1+\varphi) + \frac{\Phi(1+w_g)(1+\varphi)^2}{\Gamma}]$$
$$q_G = q_Y^{-1}[\sigma\frac{s_g}{s_c} - \sigma\frac{s_g}{s_c}\frac{\Phi w_\tau}{\Gamma}(\frac{\sigma+1}{s_c} - \frac{1}{s_d}) + \sigma\frac{s_g}{s_c}\frac{\Phi(1+w_g)}{\Gamma}(w_\tau + 1 - \frac{\sigma+1}{s_c})]$$
$$q_Z = q_Y^{-1}\sigma\frac{\Phi w_\tau}{\Gamma}\frac{s_z}{s_c}\frac{1}{s_d}$$
$$q_W = -q_Y^{-1}\sigma\frac{\Phi(1+w_g)(1+\varphi)}{\Gamma}$$

G U.S. Data

Unless otherwise noted, the following data are from the National Income and Product Accounts Tables released by the Bureau of Economic Analysis. All NIPA data are nominal and in levels.

Consumption, C. Total personal consumption expenditures (Table 1.1.5, line 2).

Government spending, G. Federal government consumption expenditures and gross investment (Table 1.1.5, line 22).

GDP, Y. $Y = C + G$.

Total tax revenues, τY. Federal current tax receipts (Table 3.2, line 2) plus contributions for government social insurance (Table 3.2, line 11) plus Federal income receipts on assets (Table 3.2, line 12).

Total government transfers, Z. Federal current transfer payments (Table 3.2, line 22) minus Federal current transfer receipts (Table 3.2, line 16) plus Federal capital transfers payments (Table 3.2, line 43) minus Federal capital transfer receipts (Table 3.2, line 39) plus Federal subsidies (Table 3.2, line 32).

Federal government debt, $Q^M B^M$. Market value of privately held gross Federal debt, Federal Reserve Bank of Dallas, http://www.dallasfed.org/research/econdata/govdebt.cfm.

Total factor productivity, A. Business sector total factor productivity, produced on 03-May-2013 by John Fernald/Kuni Natsuki, http://www.frbsf.org/economic-research/total-factor-productivity-tfp/. All published variables are log-differenced and annualized. To be consistent with model with fixed capital, we compute $dA = dY - (dhours + dLQ)$, where $dhours$ and dLQ are business sector hours and labor composition/quality actually used. Given $dA_t = 400 * log(A_t) - log(A_{t-1})$, we compute the annualized level of TFP, normalizing $A_{1947Q1} = 1$.

We use data from 1948Q1 to 2013Q1 to calibrate the model to U.S. data. For steady states, we use the sample means reported below. For the quarterly calibration, we multiply B/Y by

Variable	Mean
G/Y	0.129
τ	0.240
B/Y	0.489

4. Lump-sum transfers as a share of GDP adjust to satisfy the steady state government budget constraint.

To calibrate the exogenous processes, we apply a Hodrick and Prescott (1997) filter to time series on G_t, τ_t, Z_t, and A_t and estimate AR(1) processes using the cyclical components of the filtered data, denoting those components by $\hat{g}_t, \hat{\tau}_t, \hat{z}_t, \hat{a}_t$. Let the AR(1) be $\hat{x}_t = \rho_x \hat{x}_{t-1} + \varepsilon_{xt}$ with standard error of estimate σ_ε. Estimates appear below with standard errors in parentheses.

Variable, x	ρ_x	σ_ε
$\hat{g}_t$	0.886	0.027
	(0.029)	
$\hat{\tau}_t$	0.782	0.029
	(0.038)	
$\hat{z}_t$	0.549	0.045
	(0.051)	
$\hat{a}_t$	0.786	0.008
	(0.038)	